MATH 391 SYLLABUS

Putnam Seminar — Fall 2015

Instructor: Prof. Alex Meadows, SB 170
(240)895-4432, `ammeadows@smcm.edu`

Class: TR, 13:00–13:50, SB 161

Structure of the course: This is a weekly seminar where we will be doing problems alone and in groups. We will work on *heruistics* and *control mechanisms* for problem-solving, techniques for writing solutions, and some typical mathematics for Putnam problems.

To Get Credit:

- Come to class prepared to actively participate in problem solving, solution presenting, and class discussion.
- Spend 20 minutes each week outside of class working on Putnam problems with 1-2 partners.
- Submit three written solutions to problems. More on this later.
- Take the Virginia Tech Regional Math Contest on October 24 or do the makeup work for it.
- Take the Putnam Exam on December 5.

Let's Get Started!

Controlly advice on problem solving:

- You should always be asking yourself the following questions:
  - What am I doing?
  - Why am I doing it?
  - How will it help me?

  Asking these questions will keep you focused on the problem and its properties.
- When solving problems it is important to have lots of scrap paper. Use old fliers, backs of envelopes, blackboards, whiteboards, whatever. You don't want to be afraid of wasting paper just because you don't know what you're doing. Doing a problem helps you understand what it's asking in the first place.

- You need to be mentally flexible. Narrow-mindedness leads to disaster. Do not get locked in to one technique when others can help. Do not spend lots of time on one approach going nowhere.
- Getting stuck is fine. It is not something to get emotionally involved with. Calmly look for another path to the solution, but don't forget your first attempt.
- Problem solving can be frustrating. However, getting frustrated usually doesn't help. Remember why you're getting frustrated - You don't understand the problem. Go back, read the problem and look for clues.
- **Do not** give up easily on a problem. You might solve a problem or you might not, but one thing is certain: Sir Ender McGivvieuppie never solves the problem.

Beginning Heuristics:

1. Search for a pattern.
2. Draw a figure.
3. Choose effective notation.
4. Work backward.

Beginning Problems:

1. Assume that cucumbers are typically 99% water by weight. Suppose that 500 pounds of cucumbers are allowed to sit in the sun for an entire day. The following morning, what remains after evaporation is 98% water. What then is the weight of what remains?
2. St. Mary's has an elimination tennis tournament with $1,025$ tennis players. In each round the players pair up at random to play a match, with possibly one person not playing if the number of players is odd. The winners of each round advance to the next along with the possible odd player out. In the end there is one champion. What is the total number of matches that are played altogether in all rounds of the tournament?
3. Show that a number is divisible by 9 if and only if the sum of its digits is divisible by 9.
4. Let A be the sum of the digits of the number 4444^{4444} and let B be the sum of the digits of A. What is the sum of the digits of B?

5. A particle moving on a straight line starts from rest and attains a velocity v_0 after traversing a distance s_0. If the motion is such that the acceleration was never increasing, find the maximum time for the traverse.

6. If n is a positive integer such that $2n+1$ is a perfect square, show that $n+1$ is the sum of two successive perfect squares.

7. Show that, for any positive integer n, $1^n + 2^n + 3^n + 4^n$ is divisible by 5 if an only if n is not divisible by 4.

8. Find all differentiable functions $f : (0, \infty) \to (0, \infty)$ for which there is a positive real number a such that

$$f'\left(\frac{a}{x}\right) = \frac{x}{f(x)} \qquad \text{for all } x > 0.$$

9. In a round-robin basketball tournament with n teams $T_1, T_2, \ldots, T_n$, where $n > 1$, each team plays one game with each of the other teams and no ties can occur. Let W_r and L_r be the number of games won and lost, respectively, by team T_r. Show that

$$\sum_{r=1}^{n} W_r^2 = \sum_{r=1}^{n} L_r^2.$$