

Infinity License Exam

Carefully read for study the following questions

1. Sandy claims that $1 + 2 + 4 + 8 + 16 + \dots = -1$. Explain what is wrong with the following argument: Let $x = 1 + 2 + 4 + 8 + 16 + \dots$. Then $2x = 2 + 4 + 8 + 16 + \dots$. So

$$x = 2x - x = (2 + 4 + 8 + \dots) - (1 + 2 + 4 + 8 + \dots) = -1.$$

2. Explain why the set of natural numbers $\{1, 2, 3, 4, \dots\}$ and the set of even numbers $\{2, 4, 6, 8, \dots\}$ have the same cardinality.
 3. Show with a list that there are infinitely many natural numbers that are not prime.
 4. Using sequences, show by example that the “equation” $0 \times \infty = 0$ is not always correct.
 5. Using sequences, show by example that the “equation” $\frac{1}{0} = \infty$ is not always correct.
 6. Find the area of the region bounded by the curves $y = \frac{1}{x}$, $x = 1$, and the x - and y -axes.
 7. Explain why $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 1$.
- ∞ . How many different sizes of infinity are there?