Name: **Math 151 Spring 2013 First Opportunity**

Welcome to the first opportunity to show the professor what you've learned how to do. In the problems below, you must show all your work. Do not use the text, calculators, notes, or receive help from each other. Draw a box around your final answers. Remember to **check your work** whenever possible.

Useful formulas: $(\cos\theta, \sin\theta) = (x, y)$ on the circle of radius one at arclength distance θ counter-clockwise from the point $(1, 0)$. $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Limit Laws: Suppose that c is a constant and that the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist. Then

(1) $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

(2) $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

(3) $\lim_{x\to a} [cf(x)] = c \lim_{x\to a} f(x)$

(4) $\lim_{x\to a} [f(x)g(x)] = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$

(5) if $\lim_{x\to a} g(x) \neq 0$, $\lim_{x\to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

(6) $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$, if n is a positive integer.

We also have

(7) $\lim_{x\to a} c = c$

(8) $\lim_{x\to a} x = a$

(11) $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$, where n is a positive integer and if n is even, $\lim_{x\to a} f(x) > 0$.

1. (10 points) Explain as well as you can what a *function* is and what the *graph* of a function is.

2. (10 points) Find the domain of the function $f(x) = \sqrt{-x^2 + 8x - 12}$.

3. (5 points) State the definition of $\lim_{x \to a} f(x) = L$.

$\lim_{x \to a} f(x) = L$ means that $f(x)$ is

whenever x is

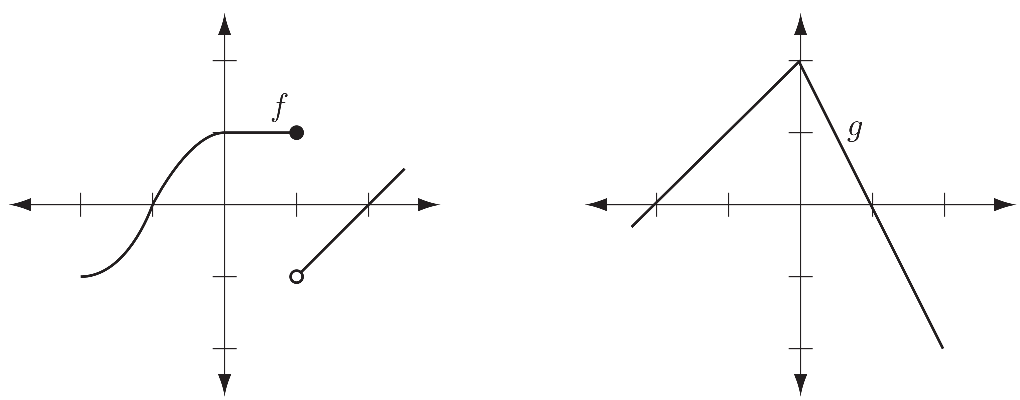
4. (15 points) **True–False:** If true, give a brief reason why. If false, use an example that shows it's false.

(a) If f is a function, then $f(1+2) = f(1) + f(2)$.

(b) $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2} = \dfrac{\lim_{x \to 2} x^2 - 4}{\lim_{x \to 2} x - 2}$.

(c) If f is a function, then $\lim_{x \to 0} f(x) = f(0)$.

5. (15 points) The figure shows the graphs $y = f(x)$ and $y = g(x)$.



$f \circ g(-1) =$

$g \circ f(-1) =$

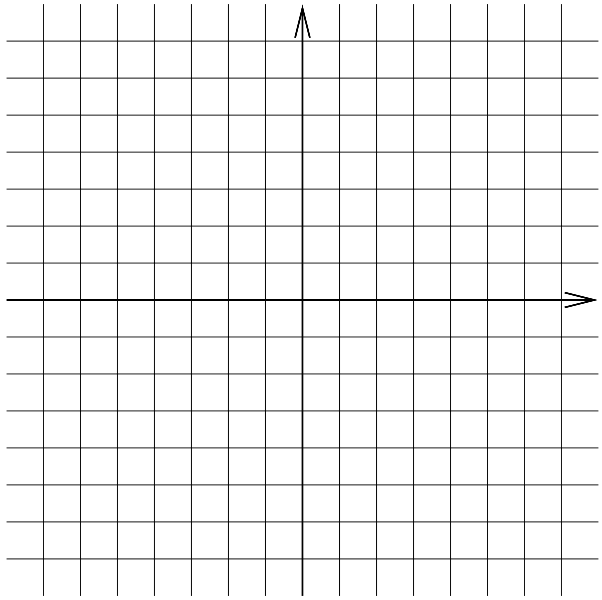
$\lim_{x \to 1^+} f(x) =$

$\lim_{x \to 1^+} f(x)g(x) =$

$\lim_{x \to 2} \dfrac{f(x) - f(2)}{x - 2} =$

6. (15 points) Use the graph paper below to sketch the graph of the function

$$f(x) = \begin{cases} x+6 & \text{if } x < -2 \\ 1-2x & \text{if } -2 < x < 0 \\ 3\sin\left(\frac{\pi}{2}x\right)+2 & \text{if } x \geq 0 \end{cases}$$



7. (15 points) The owner of the St. James Deli calculates the cost of running the shop in terms of sandwiches produced. One month, they made 1,000 sandwiches and their costs came to \$16,000. Another month, they made 1,200 sandwiches and their costs came to \$17,000.

   (a) Assuming a linear relationship, express the deli's costs as a function of the number of sandwiches produced.

(b) What is the slope of the graph of this function and what does it represent? What are the units with which the slope is measured?

8. (15 points) Yesterday I rode my bike to Leonardtown and by keeping careful track, I found that the distance I had travelled x hours after noon was exactly $f(x) = x^2 + 3x$ miles.

(a) What was my average speed between 1:00 and 3:00, in miles per hour?

(b) Use limits to compute my instantaneous velocity at 1:00. Be sure to label any limit laws that you use.

Bonus Problems: (5 points) Use the definition of the function $\sin(x)$ to explain the value of $\lim_{x \to 0^+} \frac{\sin x}{x}$.

(5 points) Determine the graph of the function in problem 2.