Math 151 Fall 2013 Practice Third Opportunity

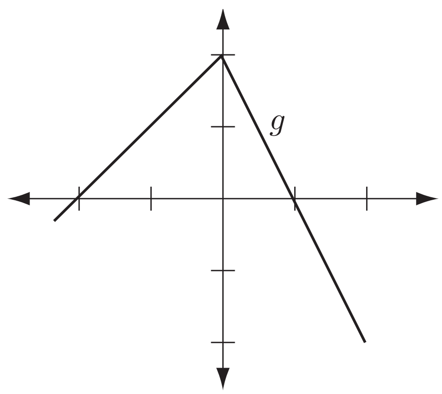
1. (a) State the Mean Value Theorem.

   (b) Draw a diagram that explains the Mean Value Theorem.

   (c) Fermat's Theorem says, "If ___ and $f'(c)$ exists, then ___." Fill in the blanks.

   (d) Name a function f and a number c such that $f'(c) = 0$, but f does not have a local maximum or minimum at c.

2. The figure below shows the graph of g, which is the *derivative* of the function f. Determine the intervals of increase and decrease, the local maxima and minima, the intervals of concavity, and the inflection points of f.



3. Princess Dido, future queen of Carthage, fled to Africa after her brother murdered her husband. There she bought for a certain amount of money as much land as she could enclose with one bull's hide. A clever mathematician, she cut the bull's hide into one long strip 100 meters in length and enclosed a rectangular piece of land along a straight shoreline of the sea of the largest possible area. What were the length and width of this rectangular piece of land?

4. (a) Approximate $\dfrac{1}{\sqrt[3]{0.97}}$.

   (b) Find the absolute maximum and absolute minimum values of $f(x) = x^2 + 2x + 3$ on the interval $[0, 3]$.

5. (a) If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?

   (b) Find the intervals of concavity and inflection points of $f(x) = x^4 - 6x^2$.

6. **True–False:**

   (a) If $f'(x) = 0$ for all x, then $f(1) = f(0)$.

   (b) If $f'(c) = 0$, then f has a local maximum or minimum at c.

   (c) Every function is continuous.

   (d) f'' is the derivative of f'.

7. Find the point on the parabola $y = 1 - x^2$ at which the tangent line cuts from the first quadrant the triangle with the smallest area.