Math 151 Spring 2013 Second Opportunity

Welcome to the second opportunity to show the professor what you've learned how to do. In the problems below, you must show all your work unless otherwise indicated. Do not use the text, calculators, notes, or receive help from each other. Draw a box around your final answers. Remember to **check your work** whenever possible.

Useful formulas: $(\cos\theta, \sin\theta) = (x, y)$ on the circle of radius one at arclength distance θ counter-clockwise from the point $(1, 0)$.

$\sin(A + B) = \sin A \cos B + \cos A \sin B$.
$\cos(A + B) = \cos A \cos B - \sin A \sin B$.

For nonzero real numbers a, b, c, $a^b a^c = a^{b+c}$, $\left(a^b\right)^c = a^{bc}$, $(ab)^c = a^c b^c$, $a^0 = 1$, $a^{-b} = \dfrac{1}{a^b}$.

Limit Laws: Suppose that c is a constant and that the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist. Then

1. $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a} [cf(x)] = c \lim_{x\to a} f(x)$
4. $\lim_{x\to a} [f(x)g(x)] = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$
5. if $\lim_{x\to a} g(x) \neq 0$, $\lim_{x\to a} \left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
6. $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$, if n is a positive integer.

   We also have

7. $\lim_{x\to a} c = c$
8. $\lim_{x\to a} x = a$

11 $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$, where n is a positive integer and if n is even, $\lim_{x\to a} f(x) > 0$.

1. (a) Use the definition of the derivative to compute $f'(2)$ where $f(x) = \sqrt{2 + x}$.
   (b) Find an equation of the line tangent to $y = \sqrt{2 + x}$ at the point where $x = 2$.

2. Suppose the line tangent to the graph $y = f(x)$ at the point where $x = 3$ passes through the points $(-2, 3)$ and $(4, -1)$.

   (a) Find $f'(3)$.

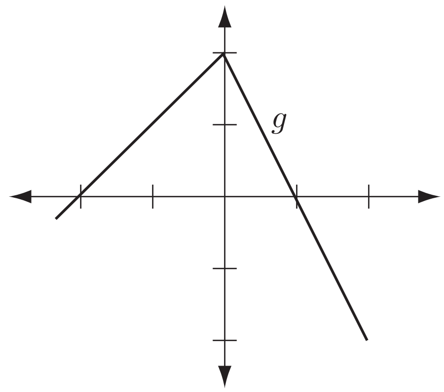
   $f'(3) = \boxed{\qquad}$

   (b) Find $f(3)$.

   $f(3) = \boxed{\qquad}$

3. Compute $\displaystyle\lim_{x \to \pi/2} \frac{2x\sin(x) - \pi}{x - \frac{\pi}{2}}$.

4. Suppose f is a function *whose derivative* is the function $g(x)$. The graph of g is shown below.



   (a) What is $f'(0)$?
   (b) What is $f''(1)$?
   (c) Let $h(x) = f(1 - 2x)$. Compute $h'(1)$.

5. Compute the derivatives of the following functions. Please do not simplify your answer. There will be very little partial credit awarded on these questions, unless you are explicit about the rules you use.

   (a) $f(x) = 5x^2 - 3x$
   (b) $g(y) = \sqrt{\sin\left(y + \sqrt{y}\right)}$
   (c) $r(t) = \dfrac{t\cos(t) + 1}{t^2 + 1}$
   (d) $h(x) = \sqrt{x\cos(2x)}$
   (e) $w(x) = \dfrac{x^3 - \tan x}{\cos x}$

6. We showed in class that if a function is differentiable at a number a, then it is continuous at that number a. Remember that f is continuous at a if $\lim_{x \to a} f(x) = f(a)$. Give an example of a function that is continuous at a number, but not differentiable at the same number. Show explicitly that the function is continuous at the point, and that it is not differentiable at the point.

   $f(x) = \boxed{}$ $\qquad a = \boxed{}$

   f is continuous at a:

   f is not differentiable at a:

7. Find a point (x_0, y_0) on the ellipse with equation $x^2 + 4y^2 = 4$ at which the tangent line goes through the point $(3, 0)$. Don't be alarmed if your answer includes a $\sqrt{}$.

8. Find a tangent line to the graph $y = x^2$ that goes through the point $(2, 3)$.