Math 151 Fall 2013 First Opportunity

Name: ____

Welcome to the first opportunity to show the professor what you've learned how to do. In the problems below, you must show all your work. Do not use the text, calculators, notes, or receive help from each other. Draw a box around your final answers. Remember to **check your work** whenever possible.

Useful formulas: $(\cos\theta, \sin\theta) = (x, y)$ on the circle of radius one at arclength distance θ counter-clockwise from the point $(1, 0)$. $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Limit Laws: Suppose that c is a constant and that the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist. Then

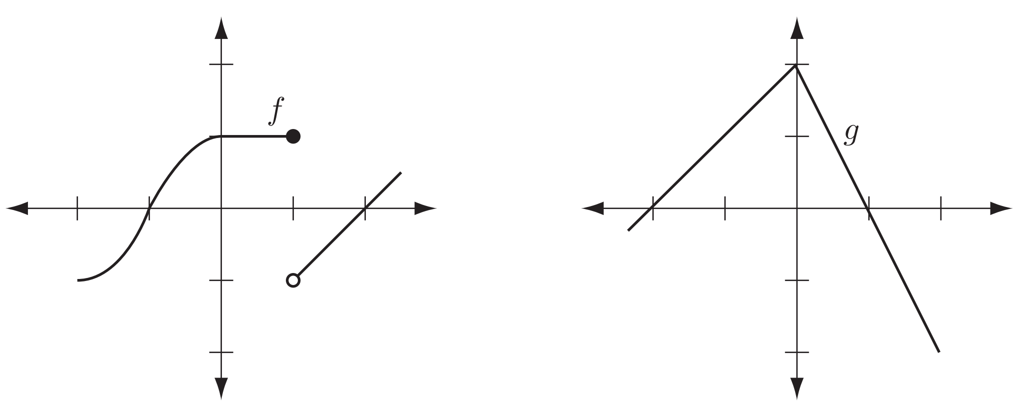
1. $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a} [cf(x)] = c \lim_{x\to a} f(x)$
4. $\lim_{x\to a} [f(x)g(x)] = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$
5. if $\lim_{x\to a} g(x) \neq 0$, $\lim_{x\to a} \left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
6. $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$, if n is a positive integer.

   We also have

7. $\lim_{x\to a} c = c$
8. $\lim_{x\to a} x = a$

11 $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$, where n is a positive integer and if n is even, $\lim_{x\to a} f(x) > 0$.

1. (25 points) The figure shows the graphs $y = f(x)$ and $y = g(x)$.



$f \circ g(-1) =$ $\boxed{}$

$g \circ g(1) =$ $\boxed{}$

$\lim_{x \to 1^+} f(x) =$ $\boxed{}$

$\lim_{x \to 0} x^2 g(x) =$ $\boxed{}$

$\lim_{x \to -2} g(x) f(x+3) =$ $\boxed{}$

$\lim_{x \to 0} g(x^2) =$ $\boxed{}$

$$\lim_{x \to 1} \frac{g(x) - g(1)}{x - 1} = \boxed{}$$

$g(x) =$ $\boxed{}$

2. (15 points) State the definition of $\lim_{x \to a} f(x) = L$.

   $\lim_{x \to a} f(x) = L$ means that $f(x)$ is $\qquad\qquad\qquad\qquad$ L

   whenever x is

   Use the definition to explain either why $\lim_{x \to 5} x = 5$ or why $\lim_{x \to 5} 3 \neq 3.2$.

3. (20 points) **True–False:** If true, give a brief reason why. If false, use an example that shows it's false.

   (a) $\sin\left(x + \frac{\pi}{2}\right) = \sin(x) + \sin\left(\frac{\pi}{2}\right)$.

   (b) If f is a function, then $f(1) \neq f(2)$.

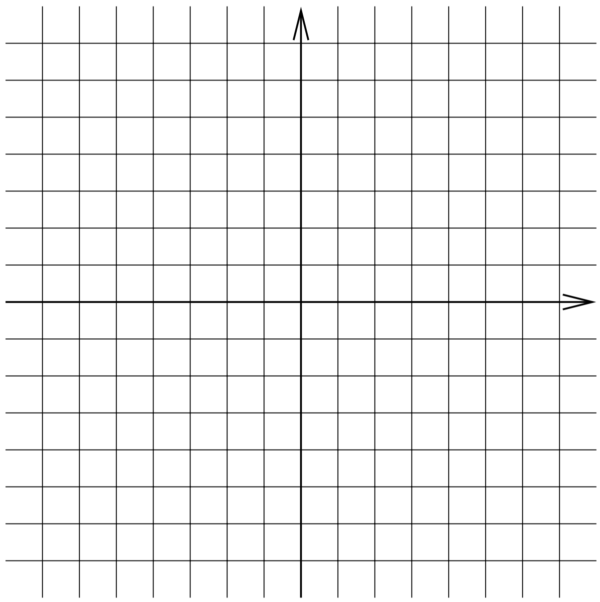
   (c) If f is a function, then $\lim_{x \to 0} f(x) = f(0)$.

   (d) If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ don't exist, then $\lim_{x \to a} [f(x) + g(x)]$ doesn't exist.

4. (15 points) Carefully use the limit laws to compute $\lim_{x \to 2} 3x^2 - x + 2$.

5. (15 points) Use the graph paper below to sketch the graph of the function

$$f(x) = \begin{cases} x+6 & \text{if } x < -2 \\ x^2 & \text{if } -2 < x < 2 \\ 0 & \text{if } x = 2 \\ 7-3x & \text{if } x > 2 \end{cases}$$



6. (15 points) The graph of the function $f(x)$ is the lower half of the circle of radius 2 centered at $(2, 0)$.

   (a) Describe the graph of $g(x) = 3 - f(x+2)$.

   (b) Find a formula for $f(x)$.

7. (15 points) Let $f(x) = \sqrt{1 - x^2}$ and $g(x) = \frac{1}{x}$.

   (a) Find $f \circ g(x)$ and state its domain.

   (b) Find $g \circ f(x)$ and state its domain.

8. (15 points) Yesterday I walked to Leonardtown and by keeping careful track, I found that the distance I had travelled x hours after noon was exactly $f(x) = \sqrt{x+3}$ miles.

   (a) What was my average speed between 1:00 and 6:00?

   (b) Use limits to compute my instantaneous velocity at 1:00.

(c) Find the equation of the line tangent to the graph $y = \sqrt{x+3}$ when $x = 1$.

Bonus Problems: (5 points) Find the slope of the line tangent to the graph $y = \sqrt{4 - x^2}$ at the point $(1, \sqrt{3})$. (You don't need to use limits)

(5 points) When $x > 0$, $x - \frac{x^3}{6} < \sin x < x$. When $x < 0$, $x < \sin x < x - \frac{x^3}{6}$. Use this information to compute $\lim_{x \to 0} \dfrac{\sin x}{x}$. (Remember: multiplying by a negative number switches the direction of $<$.)

(Credit on these problems will be used after grades have been decided. It will not be added to your exam grade. There will be little or no partial credit awarded on this problem. You are advised to make sure your answers on the main exam are correct.)