Game Theory and CS

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- Game Theory Basics
- CS and Game Theory
- Some Games
- Applications

Game Theory

- Origins in 18th century for mathematics, 1920's for modern game theory, 1950's for CS
- Mathematical models for conflict and cooperation between entities (read: agents).
- Rational agent focus.
- Core here – decision making!
- Nash equilibrium – solution concept where each player knows the strategy of the others, and changing strategies unilaterally will not gain players anything. Used for predicting outcomes.

A Handful of Types

- Some we've talked about: zero-sum, discrete, sequential, perfect information
- Combinatorial – optimal move out of a ton of possible moves.
- Differential – continuous games where evolution of state is dictated by differential equations
- Many-agent/Population
- Symmetric/Asymmetric – the payoff for a particular strategy does not depend on external factors central to agent.
- Cooperative
- Stochastic

CS & Game Theory

- Aspect of Theoretical CS
  - But a big applied CS influence!
- John von Neumann
- AI bent – making rational decisions.
- Impact on logic design, multi-agent systems, randomized algorithms.
- Techniques:
  - State-space search
  - Belief states & AND-OR graphs (stochastic)
  - Machine learning (differential)

Nim

- Potentially ancient strategy game.
- From distinct heaps, on each turn a player must remove at least one object, but can take more as long as they are all from the same heap. The player to take the last object wins.
- Theorem: the player making the first move has a winning strategy if and only if the nim-sum of the sizes of the heaps is non-zero. Otherwise, the second player has a winning strategy.

Prisoner's Dilemma

- Introduced in 1950 – probably the most used, best known game theory game.
- Two criminals are arrested, separated, and imprisoned in isolation. The police don't have enough evidence to convict either criminal on the major charge, so they will be sentenced to a year in jail on a lesser charge. The police offer each prisoner a deal – if they testify against their partner, they get to go free, and the partner goes to jail for 3 years. If they both testify against each other, they go to jail for 2 years. What should they do?
- Applications: economics, psychology, videogame trope, philosophy.

Toads and Frogs

- Played on a 1xn strip of squares, where each square is either blank, contains a toad, or contains a frog. Customarily toads occupy consecutive squares at the leftmost end, frogs occupy consecutive squares at the rightmost end.
- Left player moves a toad one square to the right if it's empty. If the square contains a frog and the square next to the frog is empty, the player may move the toad to that empty square. The player may not jump over toads, or multiple frogs.
- Right player moves a frog one square to the left, following similar rules.
- The first player to be unable to move loses.
- Can you find a strategy for one toad, one frog?
- How hard is evaluating the general case?

Chomp!

- Alex Meadow's favorite combinatorial game.
- Played on a chocolate bar – each player chooses a square to eat, and eats all squares below it and to the right.
- Player to take the last square loses… THEIR LIFE.
- Go first or second?

Exercise

- Here's a game – Starting with a heap of objects (non-zero), players take turns dividing a heap into two non-zero heaps of different sizes. The game continues until the heaps are all size 2 or smaller. The winner is the last player to make a move. Tell me, is it better to go first, or second, and you must be able to justify your answer!

Applications

- Economics – competitive behavior modeling (also in Biology), market prediction
- Social network modeling
- Operations research – equilibrium to predict demand and supply changes
- Network flow expectation
- Political Science, including WAR BARGAINING.

What's big in CS and Game Theory

- New result in iterative Prisoner's Dilemma by manipulating the other agent who is using the traditional tit-for-tat strategy.
- Search engine optimization, traffic routing
- Increasing efficiency of Nash equilibrium questions (many are NP-hard!)
- Relation of game theory and distributed computing fault tolerance.
- Implementation of intelligent mediators