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MANAGING CAPACITY AND FLOW AT THEME PARKS

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The growth of service industries and their impact on the U.S. economy have attracted considerable attention in recent years. While some service sectors, most notably airline and telecommunication industries, have been in the forefront of model development, the industry is rather fragmented, and similar rigor is lacking in most other sectors.

This paper describes an application of a model-based approach to some of the short-term ride capacity and visitor flow issues faced by the Six Flags Magic Mountain (SFMM), a major national theme park. Specifically, we consider daily operations at the theme park and focus on the generation and evaluation of alternative strategies for managing ride capacities and visitor flow. Management of demand involves two aspects: (a) understanding customer preferences as revealed by routing behavior, and (b) using the model to evaluate the implications of changes in transition-behavior.

A crucial component of the study relates to the empirical data collected. Besides verifying the validity of the models, these data provide several insights for developing schemes to manage the day-to-day operations of the park. The SFMM management was actively involved in various phases of this study and as a result has been introducing the proposed models in a phased manner.

The theme park studied here, Six Flags Magic Mountain (SFMM) located in southern California, provides a day-long total entertainment package. Theme parks possess several interesting characteristics that influence both analysis and management of their operations. First, the service package is not homogeneous—the experience includes thrill rides, shows, arcade games, and food and beverages. Second, customer preferences are not uniform, and the market could be segmented into several groups. Third, the park attendance level fluctuates significantly, according to the season, day of the week, and time of day. Fourth, customer perceptions (e.g., about delays and queues) play an important role in evaluations of the park's operations. Even more important, the interdependence between these measures is not obvious and requires insights into customers' needs and expectations.

For example, the correlation between capacity utilization and waiting time is well recognized, and the tradeoffs between capacity, operating costs, and waiting times have been addressed in a wide range of applications. However, the impact of waiting times on customer satisfaction is not very clear. Traditionally waiting has been viewed as a negative measure, and studies in the operations management literature typically assume a monotonic relationship between waiting times and customer satisfaction. However, there is some evidence to suggest that this may not be universally true in all instances in service industries. While few customers tolerate or desire long waits, it appears that in some situations customer experience and perception of service is enhanced by some waiting, and thus minimizing the waiting time is not necessarily a desirable objective. For example Larson (1987) argues that for fast food customers, satisfaction in a single-queue system may be higher than in a multi-queue system, even though customers wait longer in a single-queue system. Also, in a theme park, waiting may contribute to the experience; this notion was verified by the result of a customer satisfaction survey at the park. Although excessive waiting times are quite undesirable, low waiting times tend to have a negative impact as well. Another customer survey commissioned by the theme park suggested that beyond a threshold level of average number of rides, further rides provided little improvement in customer satisfaction. These are interesting issues and are subjects for further research.

In this paper, we describe an application of a model-based approach to some of the short-term operational issues faced by the SFMM. Specifically, we consider daily operations at the park and focus on optimally setting a ride's nominal capacity, analyzing and managing the park's visitors transition patterns, and developing models to suggest routing tours. A crucial component of our study relates to the empirical data collected. In addition to verifying the validity of the models developed, these data provide insights for improving the day-to-day operations of the park.

The remainder of the paper is organized as follows. In the next section we describe the park and discuss different customer classes and their service experiences at the park. Section 2 provides a mathematical model for managing ride capacity, a model for generating the desired transition

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pattern, for influencing future customers' movements in the park, and a model for providing a routing sequence based on specific customers preferences for the rides. Also, we develop a neural network model to estimate the observed capacity of the rides for any given nominal capacity. In Section 3 we describe the results of our data collection and related analysis. Detailed analysis of the ride capacity model, its potential for improving park operational performance, and other related managerial considerations are addressed in Section 4. In Section 5 we focus on the implementation aspects of influencing customers transition patterns in the park. We characterize transition patterns that lead to improved park performance, and propose several policies to influence visitors' behavior while they are touring the park. In Section 6 we conclude our study and discuss issues to be investigated in future work.

1. BACKGROUND

The primary attraction of the SFMM theme park lies in its thrilling roller coaster rides, which have catchy names like Viper, Colossus, and Ninja. The rides are complemented by a variety of special shows such as the Dolphin act, the U.S. high diving team, and the Batman stunt show. A wide selection of arcades, gift shops, and eating establishments completes the entertainment services provided by the park. From an operations perspective, the rides offered at the park can be classified broadly into three categories—(1) group rides, (2) continuous rides, and (3) individual rides. Colossus and Flashback are examples of rides in which customers are grouped together for a roller coaster ride. In contrast to the group rides, Metro and Orient Express are examples of continuous rides, and the ride pace is well regulated. Buccaneer and Granny Grand Prix are examples of individual rides in which the pace is less controlled.

Effective management of the rides requires clear understanding of ride capacity. Generally, a ride's nominal capacity is determined by the number of operating units (cars, boats, trains, etc.), the number of seats per operating unit, its trip time, and loading and unloading time. For example, Jet stream could be operating with either 20, 25, 28, or 32 boats. The ride cycle time is estimated to be seven minutes, with 4.5 minutes for trip time and the remainder for loading and unloading customers. Each boat can accommodate five passengers; consequently the ride nominal capacity could vary from 850 to 1360 customers per hour. Typically, the cycle time for the rides is constant, and the capacity is adjusted by altering the number of carts. Park management changes the ride capacity based on the park attendance level and queue lengths at different rides.

A ride's observed capacity may differ from its nominal capacity. For the continuous and individual rides, the observed and nominal capacities are primarily the same. But for the group rides, the observed capacity is a function of how the visitors are "grouped" and "loaded" on the operating units of the ride. Individual customers may wish to be seated alone, thus occupying a whole unit by themselves, and families and small groups may do the same thing. As a result, the ride's observed capacity may vary significantly from the nominal capacity, and for some rides it may be as low as 60% of the nominal capacity. For example, the observed capacity at Log Jammer, operating with 32 logs, has typically been around 1,200, whereas the nominal capacity is 1920 rides per hour for weekends.

The variety of special shows offered is the second class of attraction at the theme park. From an operational viewpoint these can be viewed as batch processes with the cycle time determined by the show's characteristics. The batch capacity essentially is fixed, and short-term operational decisions concern the number of shows and the corresponding schedule.

The complexity in managing the operations of the park is primarily due to the wide variation in the customers' arrival pattern and their entertainment preferences. The customers exhibit a wide variation in their preferences, and their perceptions of service can be classified into three main groups: (1) younger visitors, especially teenagers, (2) family visitors, and (3) senior citizens. The primary attraction for younger visitors lies in thrill rides, and teenagers appear to be less sensitive to long waits. In contrast, senior citizens are influenced by waiting times and tend to plan the sequence of rides in order to reduce their waiting times. Families often are constrained by height limitations that exclude certain rides. Family groups tend to have a lower tolerance for long waits than teenagers. At an aggregate level, the behavior of the three customer groups may be characterized by their transition behavior within the park, tolerance to waiting, and threshold level for the number of rides per visit.

2. MODELS FOR RIDE MANAGEMENT AT THEME PARKS

In this section we provide a set of models for managing the ride capacity and demand in the short-term operations at the park. As mentioned earlier, capacity changes are effected primarily by changing the number of operating units on each ride. However, management of ride demand involves two aspects: (1) understanding customers' preference through their transition behavior, and (2) using the model to evaluate the implications of changes in transition behavior. Thus we do not attempt to characterize the trade-offs between customer satisfaction and waiting times. Instead, we assume that a desired average number of rides is targeted (based on such trade-offs), and that the model is used to determine the optimal ride nominal capacity and to propose strategies to achieve this target. For the first level of analysis we aggregate the customer classes into one class. In addition, we focus on the rides and ignore the shows and other attractions offered by the park. Figure 1 provides a schematic of the network of rides in the park, where many small and adjacent rides have been grouped

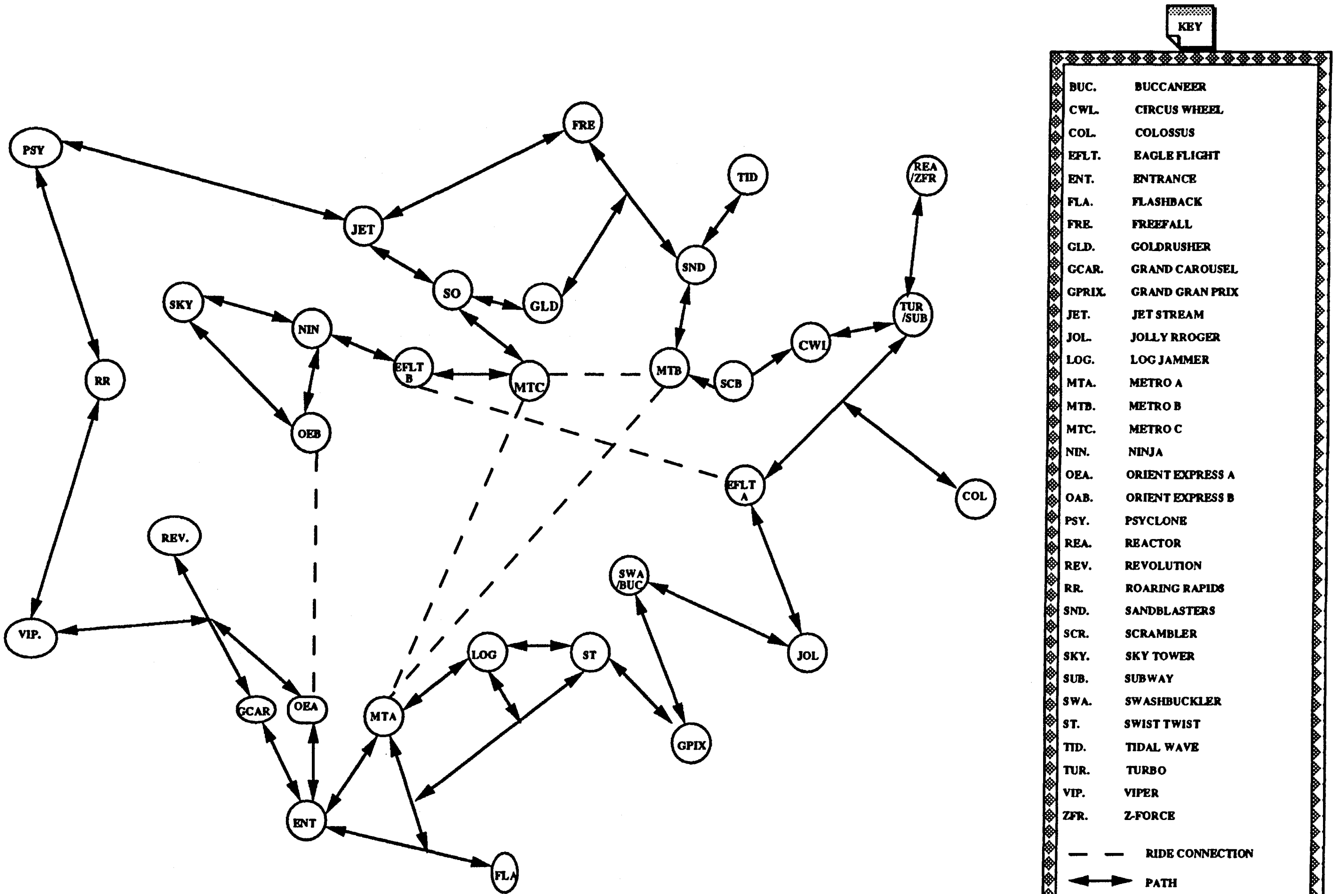

Figure 1. Network of rides.

together. Next, we describe the general characteristics of each model.

(I) A neural network model, Ride Capacity Model (RCM), is constructed to determine observed ride capacity for any group ride, where "customer grouping" is critical. In general, the neural network uses the historical arrivals and departures and the ride's nominal capacity to determine the customer grouping pattern and learn the dynamics of grouping. Based on these observations, the model estimates the ride's observed capacity.

(II) Visitors' arrival patterns at the rides, the transition patterns within the park, and the rides' observed capacity are used to determine the optimal nominal capacity level for the rides. The objective of the Capacity Management Model (CMM) is to maximize the park service level subject to park operating budget, customer threshold value for the desired number of rides, and maximum tolerable queue lengths at the rides.

(III) Existing transition patterns within the park often result in various levels of demand for different rides. This demand variation manifests itself in large fluctuations in the queue lengths for some rides at different hours. To influence the park transition pattern, we develop the Flow Pattern Model (FPM), in which the transition pattern is also a decision variable and the model simultaneously sets the ride's capacity and defines the transition within the park. Based on the "optimum" transition matrix we generate several policies to influence demands for the rides.

(IV) Finally, we construct two mathematical models to design touring plans of the park to avoid congestion. The Ride Selection Problem (RSP) decides on the set of rides with high customer utility. The Ride Visiting Problem (RVP) sequences the set of rides to be visited, given the anticipated waiting times for the rides during the day. Figure 2 shows the relationship of these models.

2.1. Ride Capacity Model

In this section, we describe the neural network model used to approximate a ride's observed capacity. The output of

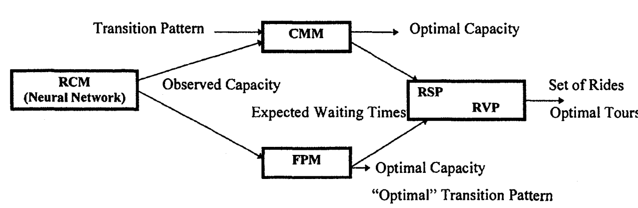

Figure 2. Overall configuration of models used.

the RCM is used in both the capacity and flow pattern models. The neural network is used to approximate the group rides, such as Colossus, Jet Stream, Log Jammer, Metro, Ninja, Psyclone, and Revolution. For the other categories of rides, the "observed capacity" was provided by the management.

We first discuss how the ride throughput (i.e., the number of customers served at each ride) is approximated. In our model, the ride throughput is estimated by the minimum of the observed ride capacity and the number of customers waiting in the ride queues, specifically:

$$S_{it} = \mathrm{Min}(Q_{it}, C_{ig}), \tag{1}$$

where S_{it} is the throughput of ride i at time period t and Q_{it} is the ride queue length. The ride observed capacity, C_{ig}, is defined by the neural network approximation of the operating characteristics of the rides. Approximation (1) was motivated by the observation that the number of rides taken was mainly based on how customers were grouped to get on the rides. But in instances when the queue length is smaller than the existing number of operating vehicle units available at the ride, the impact of customer grouping is negligible. The quality of approximation (1) is described in Section 3.

We have constructed a simple neural network for each type of ride. The networks consist of one input, one hidden layer, and one output layer which has only one node at the output layer. For further information on neural networks the reader is referred to Masson and Wang (1990) and Fort (1988). The amplification (2) and transformation (3) functions of the network may be described by the following equations:

$$U = \sum_{\lambda=1}^{\Lambda} w_\lambda X_\lambda, \tag{2}$$

$$Y = \frac{1}{(1 + e^{-\alpha U})}, \tag{3}$$

where w_λ is the weight at the output node via the connections from the hidden layer node λ, X_λ is the input from hidden layer processing element λ, and Y is the actual output of the network. The sigmoid nonlinear transformation function is given by (3), where α is a measure of noise in the system. The sigmoid function is continuous and monotone and is used in many applications. The objective of the model is to train the network such that the error in the output is minimized. Let $E = D - Y$ represent the error at the output node during the training session, where D is the desired output (the observed capacity). The data set for training the networks was collected by the data collection team. We set the change in the weights to be proportional to the negative of the derivative of the cost function, C, with respect to the connection weights, such that $\Delta w_\lambda = -\partial C/\partial w_\lambda$. From the chain rule we get:

$$\Delta w_\lambda = -\partial C/\partial w_\lambda = -\partial C/\partial Y * \partial Y/\partial U * \partial U/\partial w_\lambda. \tag{4}$$

In our model we have set $C = f(D - Y) = (D - Y)^3$, which through our experimentation was found to be quite suitable. Equation (4) determines how to change the weights along the connections in the network, where the partial derivatives are easily computed. After training the network with sample data we freeze the weights, and then for any input vector we can get the predicted output algebraically.

The neural network used in our implementation consisted of 14 nodes in the input, 13 in the hidden layer, and one node in the output layer. The nodes in the input layer correspond to the number of vehicles (one node for each capacity level), ride trip time, ride loading and unloading time, seats per unit, type of ride (batch, discrete or continuous), number of waiting lines, popularity of the ride, and a bias node. The nodes in the input and output layer are fully connected. The additional node in the input layer corresponds to the bias node, which is generally included in backpropogation networks. All the available rides data were used to train and to stop training the network, to avoid over-training. For further discussion on how to train neural network see Masson and Wang.

2.2. Capacity Management Model

The capacity management model (CMM) determines the capacity levels for the rides in the park during different time periods. The model utilizes the transition probabilities as the input parameters of the model. Notation used in the formulations is given in Table I. The CMM can be formally presented as follows:

$$\mathrm{Max} \operatorname*{Min}_{i=1}^{T} \left(\sum_{i=1}^{n} W_i S_{it} \right)$$

$$Q_{it} = Q_{it-1} + I_{it} - S_{it-1} \quad \forall (i, t), \tag{5}$$

$$I_{it} = I_{0t} P_{0ik} + \sum_{j=1}^{n} P_{jik} S_{jt-1} \quad \forall (i, k), t \in T_k, \tag{6}$$

$$S_{it} = \mathrm{Min}\left(Q_{it}, \sum_{g=1}^{G} C_{ig} Y_{igk} \right) \quad \forall (i, k), t \in T_k, \tag{7}$$

$$\sum_{g=1}^{G} Y_{igk} = 1 \quad \forall (i, k), \tag{8}$$

$$\sum_{i=1}^{n} \sum_{g=1}^{G} \sum_{k=1}^{K} |T_k| A_{ig} Y_{igk} \leq B, \tag{9}$$

$$\sum_{i=1}^{n} \sum_{t=1}^{T} S_{it} \leq (1 + \gamma) TV \sum_{t=1}^{T} I_{ot}, \tag{10}$$

$$Q_{it} \leq QX_{it} \quad \forall (i, t), \tag{11}$$

$$Y_{igk} \in (0, 1) \quad \forall (i, g, k), \tag{12}$$

$$Q_{it}, I_{it}, S_{it} \geq 0 \quad \forall (i, t). \tag{13}$$

The CMM maximizes the minimum weighted number of rides given in any time period. This measure, as discussed further in Section 3, is a more relevant measure of park performance and provides uniformity in delivery of service rather than the total number of rides given throughout the day. The objective is achieved by changing the capacity of

Table I
Notation

i, j: index of the rides, $i, j = 1, \ldots, n$
g: index for capacity level, $g = 1, \ldots, G$
t: index of time periods, $t = 1, \ldots, T$
k: index of transition patterns, $k = 1, \ldots, K$
l: index of time periods in Tour Design Model, $l = 1, \ldots, L$
F = set of pairs of rides that are far apart
T_k = a set defining the time periods in transition pattern k
Q_{it} = length of the queue at ride i at the beginning of time period t
P_{0ik} = probability of customers going to ride i upon their arrival during transition pattern k
P_{ijk} = probability of customers going to ride j from ride i during transition pattern k
F_{ijt} = number of customers going to ride j from ride i during time period t
I_{it} = number of customers arriving at ride i at the beginning of time period t
I_{0t} = number of customers arriving at the park at time period t
S_{it} = number of customers served by ride i during time period t
Y_{igk} = 1 if capacity level g is used at ride i during the transition pattern k, 0 otherwise
R_{il} = expected waiting time for ride i during time interval l given the attendance level
Z_{il} = 1 if ride i is visited in time interval 1, 0 otherwise
QX_{it} = maximum acceptable queue length, defined by management
qx_{it} = minimum desirable queue length, defined by management
C_{ig} = actual capacity of ride i operating at level g, computed through neural network model
A_{ig} = cost of operating ride i at capacity level g
W_i = customer preference associated with ride i
B = budgetary operating limit
TV = visitors' ride threshold value
γ = a safety factor for ride threshold value, $1 \geqslant \gamma \geqslant 0$

the rides at different time intervals. Constraint (5) computes the queue length at each ride for every time period. In constraint (6) we capture the movement of park visitors by the number of customers that arrive at ride i in time period t. Visitors either go directly to ride i upon their arrival at the park or join ride i from other rides based on the transition matrix. Constraint (7) determines the number of customers receiving service at ride i and time t. The ride capacity level is determined by constraint (8). Constraints (9), (10), and (11) define, respectively, the bound on the operating budget, the total rides given, and the maximum tolerable queue lengths for the rides. Constraint (10) is motivated by our empirical analysis, which indicates that customers have a threshold value for the number of rides they take during their visit to the park. Constraints (9) and (11) are managerial inputs and significantly influence the service package delivered to the park visitors.

The CMM is a mixed integer linear program of moderate size and was used in the first phase of our implementation. Details of model verification and implementation are discussed in Section 4. A variation of CMM in which arrival probabilities are also decision variables may be constructed by adding constraint (14).

$$\sum_{i=1}^{n} P_{0ik} = 1 \quad \forall\,(k)\,. \tag{14}$$

The results of this model were used as the first step in influencing the transition patterns around the park. Next we discuss the flow pattern model.

2.3. Flow Pattern Model

In the Flow Pattern Model (FPM), in addition to identifying the capacity level of each ride, we focus on capturing the desired transition probabilities and movement of visitors in the park. The FPM also seeks to find the optimum distribution of customer arrivals at the park.

$$\operatorname{Max} \operatorname*{Min}_{t=1}^{T} \left(\sum_{i=1}^{n} W_i S_{it} \right) \tag{RSP}$$

s.t.

$$Q_{it} = Q_{it-1} + I_{0t} P_{0ik} - S_{it-1} + \sum_{j=1}^{n} F_{jit} \quad \forall\,(i, k),\ t \in T_k\,, \tag{15}$$

$$S_{it} = \sum_{j=1}^{n} F_{ijt} \quad \forall\,(i, t)\,, \tag{16}$$

$$P_{ijk} = 0 \Rightarrow F_{ijt} = 0 \quad \forall\,(i, j, k),\ t \in T_k\,, \tag{17}$$

$$qx_{it} \leqslant Q_{it} \leqslant QX_{it} \quad \forall\,(i, t)\,, \tag{18}$$

(7), (8), (9), (10), (12), (14),

$$F_{ijt}, Q_{it}, I_{it} \geqslant 0 \quad \forall\,(i, j, t)\,. \tag{19}$$

The FPM maximizes the minimum weighted number of rides given in any time period. Constraint (15) identifies the queue length for various rides, at the beginning of each time period. Constraint (16) captures the conservation of flow at each ride and in each time period. Constraint (17) ensures that flows for which the existing transition probabilities are zero are forced to be zero, i.e., visitors do not travel along those links. This constraint also avoids having the visitors travel a greater distance than required to get to a ride and limits their behavior to their existing movement in the park. Constraint (18) imposes a lower and upper bound on the queue lengths for the rides. This constraint is designed to influence the visitors' perception of the rides.

Influencing the time-dependent transition patterns is a challenging and difficult task. Therefore, we used the output of the FPM model to obtain an average time independent transition matrix. This matrix can be obtained from the optimal flows along the links of the park network (F_{ijt}) and the optimum number of customers receiving service at different rides (S_{it}). The optimum transition probabilities, that generate the optimum flow according to FPM, are given by the following equation:

$$P^*_{ij} = |T|^{-1} \sum_{k=1}^{K} \sum_{t \in T_k} \frac{F_{ijt}}{S_{it}} \quad \forall\, (i, j)\,.$$

The optimum transition matrix provides a guideline for park managers to seek measures to decrease or increase the flows of visitors across different links in the park. In Section 5 we describe the implementation of this model in detail.

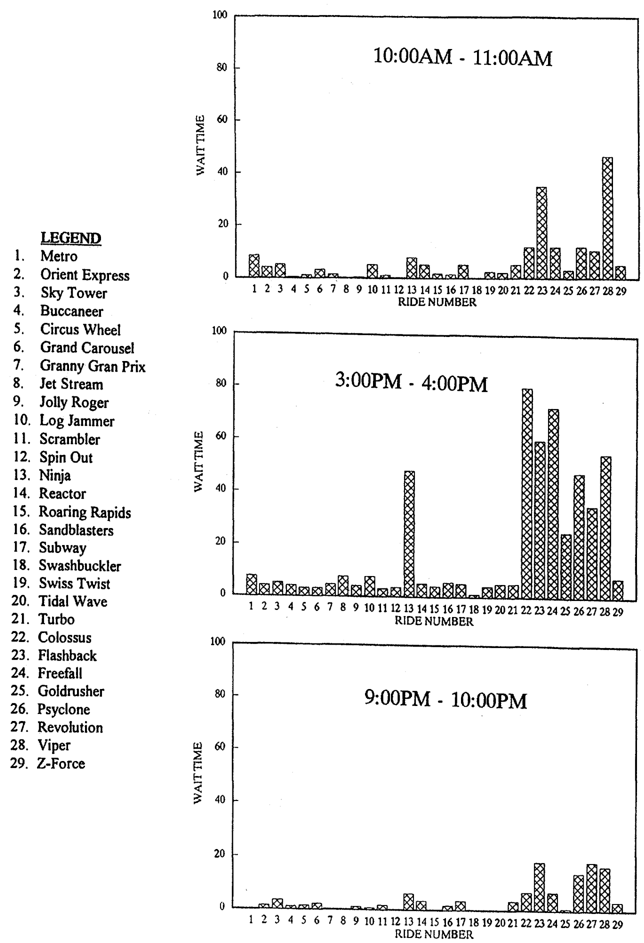

Figure 3. Ride waiting times.

2.4. Tour Design Model

Tour design is concerned with developing alternate touring plans of the park, and keeping visits as much as possible within the timeframes given and time spent waiting in the lines or traveling from one ride to another. These plans are designed to avoid congestions on days with moderate or heavy attendance. On lighter days these plans will save time, but will not be as essential to successful park visits as on crowded days. Customers experience distinct waiting times at rides during different time intervals of the day. This information is essential in designing a good tour (see Figure 3 for variations in rides' waiting times). We solve the tour problem in two stages. In the first stage we determine which rides are to be visited in each time interval. This problem is referred to as the Ride Selection Problem (RSP). In the second stage we provide a sequence for the rides to be visited, called the Ride Visiting Problem (RVP). The RSP is formulated as follows:

$$\text{Max} \sum_{t=1}^{n} \sum_{l=1}^{L} W_i Z_{il} \qquad \text{(FPM)}$$

s.t.

$$\sum_{i=1}^{n} R_{il} Z_{il} \le T_l \quad \forall\, (l)\,, \qquad (20)$$

$$\sum_{l=1}^{L} Z_{il} = 1 \quad \forall\, (i)\,, \qquad (21)$$

$$Z_{\delta l} + Z_{\mu l} = 1, \quad \delta \text{ and } \mu \in F \quad \forall\, (l)\,, \qquad (22)$$

$$Z_{il} \in (0,\, 1) \quad \forall\, (i,\, l)\,. \qquad (23)$$

The objective function maximizes the weighted number of rides visited, a surrogate for customer satisfaction. Constraint (20) decides on the set of rides that could be visited within each time interval. Based on the forecast of the attendance level, the expected waiting time for each ride (R_{il}) is computed from the CMM. In constraint (21) we define the preferred rides to be visited. With constraint (22) we impose a limit of one trip between rides that are quite far apart because their inclusion in the same time interval would force visitors to travel across the park in every time interval. This constraint would limit the travel time imposed for the rides selected.

The RSP is similar in structure to the multiple-choice knapsack problem, and the solution procedure provided by Gavish and Pirkul (1991) was modified to incorporate constraint (22), where a set of knapsack subproblems had to be solved. This approach generates many feasible solutions and effectively solves this class of problems. Solutions to the RSP define the set of rides along with the time intervals in which they have to be visited.

To complete the touring plan we need to find the ordering of the rides that minimizes the visitors' travel time. The problem of finding the ride visiting order (RVP) can be modeled as a variation of the Traveling Salesman Problem with group precedence. The rides in each period must be completed prior to the rides in the next time period.

Crucial to the implementation of our proposed solution procedure is the computation time needed to obtain a good solution. Consequently, we focus on generating fast solutions. Spacefilling curves are a natural tool to efficiently solve combinatorial optimization problems in Euclidean space (see Bartholdi and Platzman 1988). Heuristics based on this technique are quite fast in execution and are particularly suited for environments where time or computing resources are limited. An interesting feature of this approach is that clusters of points in the original space maintain their closeness in the reduced space. Combinatorial problems are much easier to solve in the reduced space. We use the following procedure to solve RVP:

Procedure to Solve RVP:

Step 1. Let the spacefilling curve θ map the unit interval onto the unit square. For the set of rides selected in the time interval 1, say N_1, compute the inverse image of rides in N_1 under θ^{-1}. Let $\theta^{-1}(N_1)$ denote the corresponding points.

Step 2. For each $1 \in L$, sort θ^{-1} (N_1)'s according to their positions on the unit interval and identify the first and last ride to be visited in each time interval. Let α_1 and β_1 denote these rides.

Step 3. Construct a layered network $G = (N, E)$ with the following characteristics

(I) Layers: The number of layers in the network is $L + 2$. Layers 0 and $L + 1$ are the source (S) and the terminal node (T).

(II) Nodes: There are two nodes in each layer 1, except the source and terminal, referred to as α_1 and β_1.

(III) Arcs: Directed arcs in the network fully connect every adjacent layer. Node S is connected to all nodes in layer 1. Node T is connected to all nodes in layer L.

(IV) Costs: Let $g(x)$ denote the cost of arc x in the network. The cost of each arc is computed as follows:

$g(S \to \alpha_1[\beta_1])$ = time to get to node $\alpha_1[\beta_1]$

$g(\alpha_1[\beta_1] \to \alpha_{l+1}[\beta_{l+1}])$ = total time needed to go from node $\alpha_1[\beta_1]$ to node $\beta_{l+1}[\alpha_{l+1}]$ to node β_{l+1} $[\alpha_{l+1}]$ and visit all the rides designated in time interval $l + 1$, starting at node $\beta_{l+1}[\alpha_{l+1}]$ and ending at node $\alpha_{l+1}[\beta_{l+1}]$.

$g(\alpha_1[\beta_1] \to \beta_{l+1}[\alpha_{l+1}])$ = total time needed to go from node $\alpha_1[\beta_1]$ to node $\alpha_{l+1}[\beta_{l+1}]$ and visit all the rides designated in time interval $l + 1$, starting at node $\alpha_{l+1}[\beta_{l+1}]$ and ending at node $\beta_{l+1}[\alpha_{l+1}]$.

$g(\alpha_L[\beta_L] \to T)$ = travel time from ride $\alpha_L[\beta_L]$ to the entrance.

Step 4. The shortest path from the source node S to the terminal node T determines the tour for visiting the rides. Step 2 of the procedure finds a Hamiltonian path for the rides in each time interval. The open tours in this step are improved by the implementation of a simple P-OPT procedure. Step 3 joins the L Hamiltonian paths optimally. In our implementation, we solve the RVP problem for each feasible solution found for the RSP. This iterative approach enables us to find the grouping of the rides that provides maximum utility, based on the preference for each ride and the total number of rides. Implementation of the tour design model is discussed in Section 5.

3. DATA COLLECTION AND ANALYSES

In this section we describe the results of our data collection and related analysis that were undertaken as a part of the study reported in this paper. The primary purpose of the empirical study was to obtain the requisite data to implement the models presented in Section 2. In addition, this phase of our study was used to develop some insights into the operations of the theme park. This additional data, some of which was qualitative, played a key role in establishing credibility with the management of the park and facilitated implementation efforts. Managerial implications of our empirical study are discussed later in this section.

To generate a data base to support our study we relied on direct observations as well as routine information collected by the park over an extended period of time. For the sake of brevity, we do not present the details of this data. Instead we provide illustrative examples and describe the results of the study. As the reader will appreciate, some of the raw data is proprietary information and is not available in the public domain.

3.1. Primary Data

Determining the transition pattern within the park was the focus of the primary data collection in this study. Specifically, a survey was administrated to 6,101 customers to obtain information related to their movements within the park. Essentially, a questionnaire elicited three pieces of information from each respondent individual or group: (a) the preceding ride visited by the customer, (b) the subsequent ride proposed to be visited, and (c) the age range of the group. The data on the subsequent ride to be visited was found to be unreliable since the customers did not always follow their plans and hence (b) was not used in the remainder of the analysis. The data on preceding ride visit was used to construct the transition matrix P_{ijk} described in Section 2. Since the survey was administered throughout the entire day, we were able to identify the time-dependent transitions. Our results indicate that it is possible to distinguish three different time-of-day-dependent transition patterns—one each in the *morning*, *afternoon*, and *evening*. The *morning* transition pattern covered three time periods, from 10 a.m. to 1 p.m. The *afternoon* transition pattern occurred until 7 p.m., and the *evening* transition covered the remaining time intervals until 10 p.m. An aggregate transition matrix of the entire day was obtained by averaging the three transition matrices. Based on the survey data, we were able to identify some dominant flow patterns with implications for the park management. These findings are discussed later in Section 3.3.

3.2. Secondary Data

The extensive data base of the park provided supplementary information for our study. The data in this data base, which was developed over a period of 20 months, between January 1990 and August 1992, may be broadly classified into two groups: ride or operations-related and customer-related data.

(a) *Ride or operations-related data.* For each of the major rides and shows, the park management collects the following information on an hourly basis: (1) ride nominal capacity, (2) hourly throughput, (3) wait times, (4) queue lengths. These data were used to define the performance functions for each ride. The empirical data were used to validate modeling efforts to capture the dynamics of rides. In our implementation we focused on two measures—throughput and queue lengths. The expressions for these two measures, given by Equation (1), are rather straightforward and were found to be acceptable for our study.

We used the above data to assess the popularity of each ride. The index for capturing customer preferences was used to define the weights W_i in the objective function of the models described in Section 2. The index was also used to develop an ABC classification of rides similar to schemes commonly found in inventory control. Based on customer preferences and ride capacity, we classified the rides into four groups. Class A, with four rides (Colossus, Viper, Ninja, and Goldrusher), accounted for 20% of the total rides. As discussed later, this qualitative insight was useful in designing implementation schemes to influence transition patterns in the park as well as in developing routing schemes for group visitors. Also, based on this classification of the rides and customer transition behavior, the park was able to evaluate the impact of closing a specific ride, relocating a ride, or adding new ride.

We were also provided with information about the variable cost of operating the rides at different capacity levels. These cost functions include the staffing, electricity, and maintenance charges. In general, three different cost functions describe the operations costs of the rides. For a subset of the rides, the operating cost is independent of the capacity level, and for other rides the operating cost varies with every capacity level.

(b) *Customer-related data.* The following data collected by the park formed a major input into our analysis and provided the necessary information to operationalize the models of Section 2: daily park attendance, hourly arrival and departure counts, number of hours park was open, and queue lengths at the rides. An important aspect of customer-related data is the distribution of customer arrivals at the park. Although the park attendance level varies significantly throughout the year and shows considerable seasonality, the percentage of total arrivals at different hours of the day is quite predictable. An empirical distribution was constructed (see Law and Kelton 1982) based on this data set and was used in the capacity and flow pattern models in Sections 4 and 5. The distribution of visitors' arrival times along with the distribution of the number of visitors in the park at any given hour was used for short-term work force scheduling of the park.

3.3. Preliminary Data Analyses and Their Implications for the Theme Park

In this section we describe our initial analyses with empirical data and discuss briefly the implications for the park management. For ease of exposition, the material has been classified into two categories.

(a) *Transition matrix, performance functions, and in-park flows.* One of the objectives of the data analysis was to validate the transition matrix and the performance of the transition functions used in the formulation of the models described in Section 2. This was achieved by observing the queue lengths and waiting times and comparing them with the results provided by the approximation functions and through the neural network model. The minimum, average, and maximum average waiting time differences were found to be 0, 6, and 14 minutes, respectively. While the sample of three days used in this experiment is too small to permit statistically significant conclusions, the results were encouraging and the park management was satisfied with the credibility of the proposed model of the functional equations.

In addition, our preliminary analysis identified dominant flow paths and gave some insights for developing implementation schemes to influence customer flows. We found that:

(i) Visitors to the park typically maintain a clockwise (60%) or counterclockwise (nearly 40%) direction for visiting the various rides/shows. This behavior was more prominent in the mornings and tended to weaken in the afternoon. The later behavior is not very surprising. Since, the most popular rides are farther along the clockwise path, and for some customers the afternoon trip is not the first one and hence they are less willing to go around the park to visit the less attractive rides/shows.

(ii) Visitors tend to remain in the same vicinity. Typically more than 50% tended to stay in the same area as the previous ride.

(iii) Within a vicinity, customers tend to visit A and B category rides first rather than the less popular C and D category rides.

(iv) A very small fraction of customers are willing to choose a ride twice in a row.

(v) There was a substantial variation (between rides and time of day) in the wait times. For example, the average wait times for the more popular rides such as Viper varied between 15 and 20 minutes in the evenings (after 7:00 p.m.) to between 40 and 55 minutes in the peak time (12:00 noon–4:00 p.m.). Charts in Figure 3 show the average wait time for the rides on Saturdays, at three different times of the day. The wait times for children's rides are small and keep decreasing until the park closes. The wait times for Freefall, Ninja, Flashback, Goldrusher, and Psyclone have their peak at mid-day. Park visitors wait the longest in the afternoon, between 2 p.m. to 6 p.m., before they can experience this group of rides. But the queues and wait times for Revolution, Viper, and Roaring Rapids are much higher in the morning than at other times during the day.

(vi) Customers do not move on from A and B rides to the other A and B rides more often than to other kinds of rides. However, certain pairs of rides show high traffic flows. For example 77% of visitors leave Turbo for Subway, 73% leave Orient Express for Sky Tower, 62% go from Grand Carousel to Log Jammer, and 53% of visitors at Swashbuckler go to Colossus. Other significant traffic flow can be derived from the aggregate transition pattern. In no case did a significant traffic flow occur between two C and D rides in different areas of the park. Also, more people prefer to ride on Viper immediately after the park opens than any other ride. Finally, few people are willing to ride on the same A and B ride twice in a row, indicating that only a small proportion (less than 6.8%) of the customers would repeat a long line after enduring the first one.

(vii) As discussed in Section 1 and indicated by the park management, operating profits are positively correlated with the duration of visitors' in-park stay. But the average in-park stay does not continuously increase with the number of rides given per person per day. Further analysis of average in-park stay indicates that there is a threshold value for average number of rides of about 12 rides per person per day. Although this ride threshold value could not be reached on many crowded days, on days when it could be reached and surpassed, customers chose not to stay any longer in the park, indicating that the extra benefit of an additional ride is almost nil, except in customer perception of the park. Also, an increase in the total number of guests attending the park tends to increase the length of in-park stay, since it will take customers longer to reach their ride threshold or, otherwise put, to "get their money's worth." When expecting high customer attendance, the park management can also increase the duration of the park's open hours to provide a higher level of service to visitors.

(b) *Park performance measure.* The Operations Department within the theme park corporation uses "number of rides per person per day," to measure park performance. This is the industry standard for determining the operating efficiency of all theme park locations. Our preliminary analysis indicated the deficiencies of this performance measure. Theme parks operate in different environments with different customer demographics, various park schedules, and different lengths of open season. Also, distinct customer classes may have different ride threshold values; thus, merely counting the number of rides given is seldom a sole measure of customer satisfaction. The problem of service measurement is further complicated by noticing that many service systems do not experience a homogeneous input and thereby may not have a single crucial measure by which to evaluate their performance. Some of our findings in this respect can briefly be described as follows:

(i) Customers tend to spend more time in the park—about 30 minutes—for every additional hour that the park is open.

(ii) The average number of hours spent in the park increases by 1.4 hours for every increase of one in the logarithm of the total number of guests.

(iii) The rides/person/hour decreases with the logarithm of the total number of guests.

(iv) Average number of rides per person per hour is a better measure of customer satisfaction than average number of rides per person per day, and consequently a better measure of efficient park operation. This measure motivated the objective function for the models in Section 2.

These findings served as a basis for achieving improved operational performance at the park. The models constructed in Section 2 were also influenced by these analyses. The ride threshold value and average ride per person per hour were used by the park management to construct a new tracking system for customer satisfaction. The analysis in this section has led to the search for a more definitive measure of service performance, to include other dimensions of the service package delivered by the park.

4. MANAGING RIDES CAPACITY

Poor management of theme park capacity may result in considerable *undesirable* customer waiting or underutilization of the available capacity. Capacity management at the theme park is exacerbated not only by low utilization of some rides but also by lengthy queues for other rides, during distinct hours of the day. Customers also have varying degrees of tolerance for waiting for thrill rides during different hours of the day—adding further to the complexity of park management. In this section we discuss different aspects of ride capacity management at the park. These analyses include discussion of various policies and their implications for improving theme park performance. First we verify the quality of the models developed.

4.1. Model Verification and Potentials

In Section 3 we established the validity of the approximation functions and verified the quality of the transition

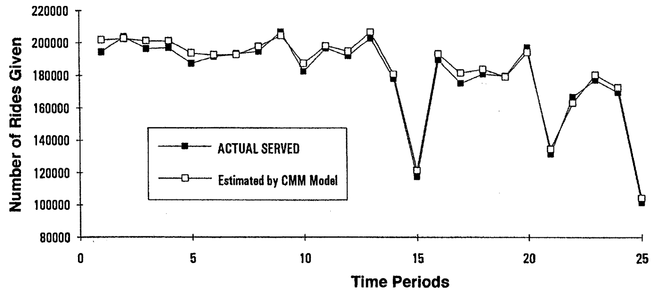

Figure 4. Model's prediction vs. actual performance.

functions used in our mathematical models. To verify the CMM's potential contribution empirically, we extracted hourly information on actual capacity levels and number of rides given throughout the period from February 22, 1993 to August 8, 1993. The attendance level for these days ranged from 9,261 to 35,646 and the cumulative number of rides given in one day (excluding the shows) ranged from 101,518 to 206,724. We simulated the park environment by using the same arrival patterns, park operating hours, and capacity levels used during the day for each ride. Figure 4 compares the actual service provided versus the model predictions. The average error was +0.79% and the maximum error was +2.75%. Although the model overestimates the park performance, the magnitude of percentage of error is well within the acceptable range. This step of model verification provided further justification for using three transition matrices throughout the day to capture the movement of customers while in the park.

In addition to verifying the model's prediction power, we also had to establish the degree of improvement that management could expect to obtain from implementing the CMM, instead of the existing ad hoc way of changing the ride capacity level. Additionally, we performed an experiment in which the operating budget and the maximum queue lengths at various rides were forced to be similar to the experiment discussed at the beginning of this section, while the capacity was allowed to change with the changes in the customer transitions intervals. The result shows that, on average, a 6.89% improvement was gained by the CMM, ranging from 2.6% to 15.2% over the current myopic practice. Adjusting for the model's average overestimating tendency of 0.79%, the improvement amounts to 0.87 additional ride per person per day. To place the CMM's improvement in the appropriate context, we derived lower and upper bounds on park performance by setting the operating budget constraint (9) to minimum and maximum levels possible. The average improvements amounted to 37% of the gap between the upper and lower bound values for the test data. Figure 5 depicts the improvements.

4.2. Capacity Management Policies:

Three capacity planning strategies were examined. These strategies vary in terms of ride capacity usage and imply different work-force scheduling for operating the rides. The policies are referred to as *static*, *dynamic*, and *flexible*

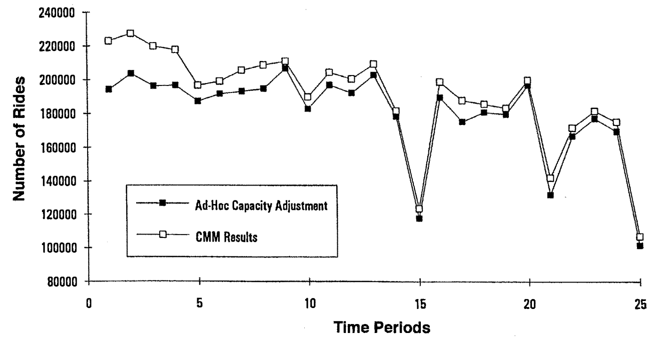

Figure 5. Expected improvement from CMM.

policies. In the *static* policy we attempt to obtain the optimal—but constant—ride capacity throughout the day. In this version of the model, constraint (8) is replaced with (24).

$$\sum_{g=1}^{G} \sum_{k=1}^{K} Y_{igk} = 1 \quad \forall\, (i)\,. \tag{24}$$

The obvious advantage of this policy is its stable work-force requirement for operating the rides. For the *dynamic* policy, K, the number of time intervals in which the ride capacity could change (in constraint (9)) was set to three to correspond to the changes in the number of customer transition patterns. Although this policy requires variable work-force sizes, the schedules could be easily accommodated within the existing park operating performance and/or by redeployment of operators among the rides. In the *flexible* policy, K was set equal to T (the number of hours the park was open), to evaluate the extent of ride capacity changes during the day.

We simulated these alternative scenarios for different attendance levels at the park. The attendance level was changed from 10,000 to 34,000 to provide a larger and more uniform spectrum of attendance statistics. The empirical distribution of percentage of arrivals was used to get the estimated hourly arrivals. Furthermore, the operating budget, which was set by the park management, was approximately 75% above the minimum operating budget needed, and the queue lengths were limited by the maximum queue lengths found in Section 4.1. Figure 6 plots the performance of various policies. The results have several significant managerial implications:

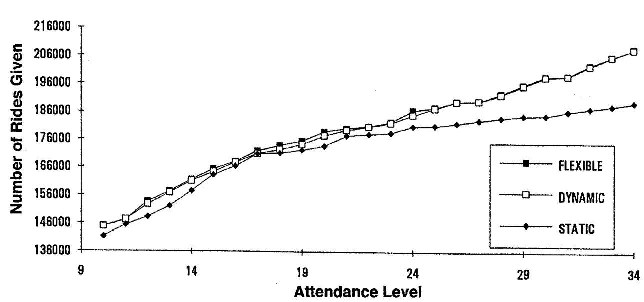

Figure 6. Comparison of different capacity policies.

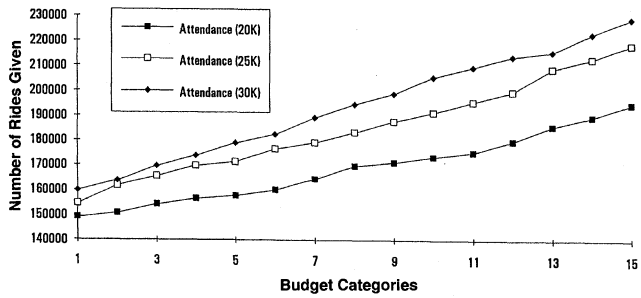

Figure 7. Service-budget trade-off curves.

(i) With low park attendance—less than 15,000—park visitors reach their ride threshold values with the park simply operating at minimum capacity level.

(ii) With high levels of attendance—more than 27,000—the park needs to operate at maximum capacity level. In addition, show schedules have to be increased to achieve the desired customer service level.

(iii) Maximum benefit from CMM is achieved in the range of 15,000 to 27,500 customers attending the park. This range covered 42% of the days in our data set.

(iv) The performances of *dynamic* and *flexible* policies are quite close. The *flexible* model only results in 0.29% additional improvement. The *static* model causes 4.85% lower performance.

The operating budget in the preceding simulation was based on the park management's decision. To evaluate the impact of this managerial decision on park performance, we developed a budget-service trade-off curve, which provides an estimate of the expected service level delivered for different levels of total operating budget. Figure 7 shows the budget-service plot. The trade-off curves are shown for three different attendance levels—15,000, 20,000, and 25,000—for which the capacity management model was most effective. These trade-off curves were crucial in bringing the park service delivery closer to customer expectation and desired threshold value, since the park managers could evaluate the impact of budget allocation on park performance.

4.3. Implementation Aspects

The park attendance level varies significantly throughout the year and shows considerable seasonality. The disparity in forecast and actual attendance level complicates the implementation of the CMM. Although the average forecast error was negligible, the forecasting system currently in place results in significant forecast errors, either in overestimating or underestimating the actual attendance. This observation was compelling enough to abandon the idea of setting ride capacity based solely on the park's forecast of attendance level. Instead we developed a *hybrid* policy that uses both the forecast of arrival for the day and the actual information about the visitors' arrival up to 11 a.m.

The *hybrid* policy was motivated by the observation that total park attendance is highly correlated with the total arrivals in the first two hours. The squared correlation coefficient was 0.896. We developed a regression model to obtain an updated estimate of the cumulative arrivals, which was used to readjust the ride capacities for the remainder of the day. Estimates of remaining hourly arrivals were obtained by using the empirical distribution of percentage of arrival during the day. Correspondingly, the capacity model had to be run twice to set the ride capacity levels, once prior to the opening of the park and then after the statistics of the second hour were available.

Lindo's (1992) industrial optimization package was used to solve all the resulting mixed integer programs in our experimentations. The average time for solving the problems examined was 18 minutes (excluding the generation of input matrix), and the maximum time was 37 minutes. All the problems were solved on HP Vectra 486 machine with 32-bit processor and 66-MHZ speed.

5. MANAGING FLOWS IN THE PARK

In Section 3 we described the existing transition patterns and concentrated on identifying how visitors move throughout the park. We identified three dominant transition patterns and analyzed their implications for the theme park. In this section we focus on characterizing the *optimum* transition patterns and develop managerial policies to influence customer behavior in order to improve the park's service level.

5.1. Optimum Transition Patterns

The FPM developed in Section 2 is used to capture the desired transition probabilities and movement of visitors in the park. The FPM, in addition to identifying the optimum transition patterns, also configures the capacity level at each ride. To make the results amenable to implementation, the following measures were taken. The queue length of each ride was restricted to 80% of the maximum queue length observed at different attendance levels. This constraint reflects the visitors' waiting behavior and the theme park's desire to reduce the visitors' waiting times. Also, the flow patterns in the park were restricted to the routes taken by the customers, so that they would not have to travel long distances to get to their next rides. The effect of this constraint is to limit the managerial measures to efforts at increasing or decreasing the flow of visitors within the present customer transition patterns. The budgetary constraints and the rides threshold were defined by the park managers.

To understand the extent of the optimum transition patterns' influence on park performance and number of rides given, we plot the results of our simulation in which the attendance level at the park was varied from 10,000 to 34,000. We also report the results obtained from the revised version of the CMM, in which the arriving transition probabilities are decided in addition to the ride capacities. These results are compared with the results obtained from

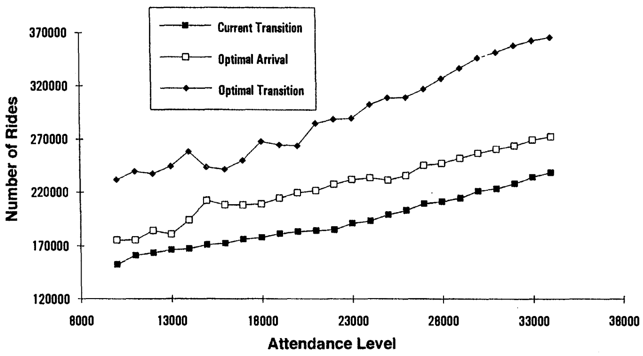

Figure 8. Expected improvements from flow management.

the CMM, which would indicate the additional improvement expected over and above what the CMM model would provide. In all the models the budgetary limits were the same and time-of-the-day transition patterns were similar to the transition intervals identified in Section 3. Figure 8 depicts the results obtained. On average, a 17% improvement over the optimum results obtained from the CMM was obtained. However, 57% improvement could be expected by alternating the overall flow patterns. The aggregate transition flow is obtained over different attendance levels ranging from 15,000 to 25,000. Comparing the current and the desired transition patterns identified the paths along which the flow has to be adjusted to achieve the preferred behavior. Some of the significant flows to be increased are: Spin-out ⇒ Roaring Rapids, Swashbuckler ⇒ Scrambler, Roaring Rapids ⇒ Revolution, and Orient Express ⇒ Spin-out. The major flows to be decreased are: Z-force ⇒ Reactor, Orient Express ⇒ Sky Tower, Grand Prix ⇒ Flashback, and Psyclone ⇒ Jet Stream.

With the current transition patterns, the customers' ride threshold value could be reached when the attendance level is less than 15,000. With the optimum arrival patterns, the ride threshold values could be attained with up to 19,000 customers arriving at the park. Given the optimum movement within and upon arrival at the park, the ride threshold value was attained at all the various attendance levels. With a relaxed operating budget and the new arrival patterns, the ride threshold could be reached up to the 25,000 level, indicating the need to influence the flow patterns within the park.

5.2. Policies to Influence Transition Patterns

Several policies were designed to induce visitor behavior toward the optimum patterns. In this section we present one of these options in detail and address its likely impact the park performance.

The scheduling (number and timing) of the shows and other entertainment in the park is crucial to park performance. Although they create additional dimensions to the service package offered by the park, these activities are designed to improve the operational efficiency of the park and alleviate the loads on the rides with long queue lengths during the day. Generally, shows and theatrical attractions are viewed as less attractive and popular than the thrill rides offered by the park. The longest wait for a show usually does not exceed the duration of one complete performance. In addition, our interviews indicated that many visitors defer their plans to attend the shows until later in the afternoon, hoping to use the early part of the day for the most attractive rides. This observation motivates the scheduling of the shows with higher frequencies in the afternoons from 2 p.m. to 6 p.m. Colossus, Ninja, Flashback, Goldrusher, and Psyclone experience their longest queues during this time interval.

Shows attract visitors away from the crowded rides, in effect reducing park congestion levels. To appraise the likely impact of the shows and generate plausible schedules, we modified the transition probabilities accordingly to accommodate the shows. Given the proposed show schedules in any time period, the functional equations provide new estimates of the queue lengths at major rides. Developing an optimization model that yields the optimum schedule and frequency of the shows is part of our ongoing research.

Additionally, the communication devices (TV monitors, signs, etc.) in the park could be used favorably to alter the existing transition patterns. TV monitors at each location could provide information about waiting time, show schedules, and indicate rides with shorter queue lengths. Signs posted at the waiting line areas could indicate the approximate waiting time to deter customers who have low tolerances for experiencing long waiting times. This practice points toward a different queue management: rather than hiding the actual queue length, which makes the wait more uncertain, reduce the visitor's anxiety by providing reliable information about expected waiting time, thereby reducing stress while waiting (Maister 1984). Lastly, moving attractions such as magicians and hypnotists could also direct customers toward less congested parts of the park.

5.3. Designing Planning Tours

Until recently, the theme park provided few suggestions for visitors to better plan their tour of the park. These suggestions basically indicated the peak hours for water rides, coasters, restaurants, shows, and gift shops. These recommendations fell short of a comprehensive touring plan. The tour design models (RSP and RVP) have stimulated the park management to seek ways to operationalize of these procedures for all park visitors. These models are now in use for generating touring schedules for guided group tours, which are provided by the park.

The touring problem was used to create various alternative tours for different categories of visitors, where tolerance to waiting time, preference for rides, and height limitations, as well as the length of the tour, were taken into account.

6. CONCLUSIONS AND FURTHER RESEARCH

We have described a preliminary application of a model-based approach to managing the capacity and flow at theme parks. The management of the park was actively involved in various phases of this study and provided considerable guidelines for shaping the direction of the research.

An interesting research aspect of theme parks and other service systems in which customers experience long queues lies in understanding different aspects of customer perceptions of the waiting times. It would be particularly interesting to explore how the capacity and flow management problems should be modeled, such that visitors' perception of acceptable waiting times as well as the throughput of the rides is incorporated. Such analysis requires additional data on visitor perceptions of ride waiting times in order to improve customer satisfaction with the service delivery system.

Finally, the analysis presented in this paper has used a single customer class and we did not incorporate the demography of the customers in the park transition patterns. Our questionnaire included the age categories of the customers, but further data regarding the distribution of different customer classes at different times of the year need to be collected to facilitate extension of our models.

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REFERENCES

BARTHOLDI, J. J. AND L. K. PLATZMAN. 1988. Heuristics Based on Spacefilling Curves for Combinatorial Problems in Euclidean Space. *Mgmt. Sci.* **34,** 3, 291–305.

FORT, J. C. 1988. Solving a Combinatorial Problem Via Self-Organizing Process: An Application of the Kohonen Algorithm to the Traveling Salesman Problem. *Biological Cybernetics*, 33–40.

GAVISH, B., AND H. PIRKUL. 1991. Algorithm for the Multi-Resource Generalized Assignment Problem. *Mgmt. Sci.* **37,** 6, 695–713.

LARSON, C. R. 1987. Perspective on Queues: Social Justice and the Psychology of Queues. *Opns. Res.* **35,** 6, 895–905.

LAW, A. W. AND W. D. KELTON. 1982. *Simulation Modeling and Analysis*. McGraw Hill.

LINDO. 1992. *Lindo: Optimization Modeling Language*. LINDO System, Inc., Chicago, IL.

MAISTER, D. H. 1984. *The Psychology of Waiting in Lines*. HBS Note 9-684-064.

MASSON, E. AND Y. WANG. 1990. Introduction to Computation and Learning in Artificial Neural Network. *EJOR,* **47,** 1–28.