The Generalized Assignment Problem and Its Generalizations

Mutsunori Yagiura[†] and Toshihide Ibaraki[‡]

[†]Graduate School of Information Science, Nagoya University
Furocho, Chikusaku, Nagoya 464-8603, Japan

[‡]The Kyoto College of Graduate Studies for Informatics
7 Monzencho, Tanaka, Sakyo-ku, Kyoto 606-8225, Japan

yagiura@nagoya-u.jp, ibaraki@ieee.org

The generalized assignment problem is a classical combinatorial optimization problem that models a variety of real world applications including flexible manufacturing systems [6], facility location [11] and vehicle routing problems [2]. Given n jobs $J = \{1, 2, \ldots, n\}$ and m agents $I = \{1, 2, \ldots, m\}$, the goal is to determine a minimum cost assignment subject to assigning each job to exactly one agent and satisfying a resource constraint for each agent. Assigning job j to agent i incurs a cost of c_{ij} and consumes an amount a_{ij} of resource, whereas the total amount of the resource available at agent i is b_i. An assignment is a mapping $\sigma\colon J \to I$, where $\sigma(j) = i$ means that job j is assigned to agent i. Then the *generalized assignment problem* (GAP) is formulated as follows:

$$\begin{aligned} &\text{minimize} && cost(\sigma) = \sum_{j \in J} c_{\sigma(j),j} \\ &\text{subject to} && \sum_{\substack{j \in J \\ \sigma(j)=i}} a_{ij} \le b_i, \quad \forall i \in I. \end{aligned} \tag{1}$$

The GAP is known to be NP-hard, since the partition problem of a given set of positive integers into two equal sized subsets can be reduced to GAP with $m = 2$. Researchers have studied the problem since the late 1960s, and computer codes for practical applications emerged in the early 1970s.

Yagiura, Ibaraki and Glover [12] proposed a tabu-search algorithm for GAP. It features an ejection chain approach, which is embedded in a neighborhood construction to create more complex and powerful moves. It also incorporates an adaptive mechanism for adjusting search parameters, to maintain a balance between visits to feasible and infeasible regions. Computational comparisons are conducted on benchmark GAP instances known as types C, D and E. These test problems are taken from the OR-Library, which is the primary repository for such problems, and are supplemented by additional test instances generated by the authors. Computational results on small instances with up to 60 jobs show that the proposed algorithm obtains solutions that are optimal or that deviate by at most 0.16% from the best known solutions.

Comparisons with other approaches from the literature show that, for instances of larger sizes with up to 1600 jobs, it obtains the best solutions among all heuristics tested. This algorithm was further improved by the authors using the path relinking approach [13]. The results show that the path relinking GAP method not only improves the previous best approach, but is especially effective for the types D and E instances, which are known as the most difficult ones among existing benchmarks.

Motivated by practical applications, various generalizations of GAP have been proposed. The *multi-resource generalized assignment problem* (MRGAP), in which more than one resource constraint is considered for each agent, is a natural generalization of GAP and has many practical applications, e.g., in distributed computer systems and in the trucking industry [3, 4, 8]. For MRGAP, Rego et al. [10] applied a metaheuristic approach related to tabu search called RAMP (relaxation adaptive memory programming)[9], and Yagiura et al. [14] devised a tabu search algorithm incorporating very large-scale neighborhood search.

Components in the algorithms proposed for GAP and MRGAP in [12, 13, 14] are further generalized by Yagiura et al. [15] to make them applicable to wider range of problems. They consider the *multi-resource generalized quadratic assignment problem* (MR-GQAP), which is a natural generalization of the *generalized quadratic assignment problem* (GQAP) [1, 5] and MRGAP. In addition to cost and resource coefficients, MR-GQAP involves a cost matrix between jobs and cost matrix between agents. The objective is to find a minimum cost assignment of jobs to agents subject to cardinality constraints and multi-resource constraints for each agent, in which the following two types of costs are considered: One is individual cost associated with each assignment, and the other is mutual cost associated with each pair of assignments. The MR-GQAP is very general and includes such problems as the graph coloring problem, a special case of the channel assignment problem, and so forth. It is also motivated by some problems emerging from real-world applications such as production scheduling problems in steal industry.

The algorithm PR-CS proposed in [15] was compared with a general mixed integer programming solver CPLEX 9.0.0 and a general solver for the constraint satisfaction problem by Nonobe and Ibaraki (denoted NI)[7], on randomly generated MR-GQAP instances with up to 200 jobs. These instances are very difficult and CPLEX was not able to find their feasible solutions even after long computation time. PR-CS succeeded in finding better solutions than NI for 44 instances out of 45 even though NI was allowed longer computation time. PR-CS was then applied to GQAP (a sepcial case of MR-GQAP) and compared with an existing algorithm MA [1] by Cordeau et al., sepcially tailored to GQAP. PR-CS obtained solutions with exactly the same objective values with MA for all 21 instances within a comparable (or even smaller) computation time. In view of its generality, these results of PR-CS appear quite promising.

References

[1] J. Cordeau, M. Gaudioso, G. Laporte and L. Moccia, A memetic heuristic for the generalized quadratic assignment problem, *INFORMS Journal on Computing*, 18 (2006) 433–443.

[2] M.L. Fisher and R. Jaikumar, A generalized assignment heuristic for vehicle routing, *Networks*, 11 (1981) 109–124.

[3] B. Gavish and H. Pirkul, Allocation of databases and processors in a distributed computing system, in: J. Akoka, ed., *Management of Distributed Data Processing*, North Holland Publishing Company, Amsterdam, 1982.

[4] B. Gavish and H. Pirkul, Computer and database location in distributed computer systems, *IEEE Transactions on Computers*, 35 (1986) 583–590.

[5] C. Lee and Z. Ma, The generalized quadratic assignment problem, Technical Report. Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada, 2003

[6] J.B. Mazzola, A.W. Neebe and C.V.R. Dunn, Production planning of a flexible manufacturing system in a material requirements planning environment, *International Journal of Flexible Manufacturing Systems*, 1/2 (1989) 115–142.

[7] K. Nonobe and T. Ibaraki, A tabu search approach to the CSP (constraint satisfaction problem) as a general problem solver, *European Journal of Operational Research* 106 (1998) 599–623.

[8] H. Pirkul, An integer programming model for the allocation of databases in a distributed computer system, *European Journal of Operational Research*, 26 (1986) 401–411.

[9] C. Rego, RAMP: A new metaheuristic framework for combinatorial optimization, in: C. Rego and B. Alidaee, eds., *Metaheuristic Optimization via Memory and Evolution: Tabu Search and Scatter Search*, Kluwer Academic Publishers, pp. 441–460, 2005.

[10] C. Rego, L. Sagbansua, B. Alidaee and F. Glover RAMP and primal-dual RAMP algorithms for the multi-resource generalized assignment problem, Research Report, Hearin Center for Enterprise Science, School of Business Administration, University of Mississippi, USA, 2005.

[11] G.T. Ross and R.M. Soland, Modeling facility location problems as generalized assignment problems, *Management Science*, 24 (1977) 345–357.

[12] M. Yagiura, T. Ibaraki and F. Glover, An ejection chain approach for the generalized assignment problem, *INFORMS Journal on Computing*, 16 (2004) 133–151.

[13] M. Yagiura, T. Ibaraki and F. Glover, A path relinking approach with ejection chains for the generalized assignment problem, *European Journal of Operational Research*, 169 (2006) 548–569.

[14] M. Yagiura, S. Iwasaki, T. Ibaraki and F. Glover, A very large-scale neighborhood search algorithm for the multi-resource generalized assignment problem, *Discrete Optimization*, 1 (2004) 87–98.

[15] M. Yagiura, A. Komiya, K. Kojima, K. Nonobe, H. Nagamochi, T. Ibaraki and F. Glover, A path relinking approach for the multi-resource generalized quadratic assignment problem, Proc. Engineering Stochastic Local Search Algorithms—Designing, Implementing and Analyzing Effective Heuristics (SLS 2007), Brussels, Belgium, September 6–8, 2007, Lecture Notes in Computer Science 4638, pp. 121–135.