COSC 480/MATH 482
Example Big M Walkthrough
Fall 2012

Here's the problem:

Minimize: $Z = 0.4x_1 + 0.5x_2$
Subject To: $0.3x_1 + 0.1x_2 <= 2.7$
$0.5x_1 + 0.5x_2 = 6$
$0.6x_1 + 0.4x_2 >= 6$
With: $x_1, x_2 >= 0$

The first thing we'll need to do is to convert this to an augmented form. First step, adding a slack variable to the first constraint:

$0.3x_1 + 0.1x_2 + x_3 = 2.7$

Then, we start to deal with the equality in the second constraint by using the big M method. First, we add an artificial variable to the second constraint:

$0.5x_1 + 0.5x_2 + x_4 = 6$

and add it to the Z function:

$Z = 0.4x_1 + 0.5x_2 + Mx_4$

Next, we deal with the third constraint by subtracting a surplus variable, then adding an artificial variable.

$0.6x_1 + 0.4x_2 - x_5 + x_6 = 6$

we also add $M * x_6$ to our Z function:

$Z = 0.4x_1 + 0.5x_2 + Mx_4 + Mx_6$

Then, we negate our Z function to switch from minimize to maximize:

$-Z = -0.4x_1 - 0.5x_2 - Mx_4 - Mx_6$

Because we can't have x_4 or x_6 in our Z function, we need to eliminate it by first solving for x_4 using the second constraint, then subbing it back in to the Z function (note that we've adjusted the Z function to have a constant on the right hand side):

$x_4 = 6 - 0.5x_1 - 0.5x_2$
$-Z + 0.4x_1 + 0.5x_2 + M(6 - 0.5x_1 - 0.5x_2) + Mx_6 = 0$

Which we then simplify to:

$-Z - (0.5M - 0.4)x_1 - (0.5M - 0.5)x_2 + Mx_6 = -6M$

Now, we do the same thing with x_6:

$x_6 = 6 - 0.6x_1 - 0.4x_2 + x_5$
$-Z - (0.5M - 0.4)x_1 - (0.5M - 0.5)x_2 + M(6 - 0.6x_1 - 0.4x_2 + x_5) = -6M$

Which we then simplify to:

$-Z - (1.1M - 0.4)x_1 - (0.9M - 0.5)x_2 + Mx_5 = -12M$

So, we are looking at the following, before we start messing with our tableau:

(0) $-Z-(1.1M-0.4)x_1-(0.9M-0.5)x_2+Mx_5=-12M$
(1) $0.3x_1+0.1x_2+x_3=2.7$
(2) $0.5x_1+0.5x_2+x_4=6$
(3) $0.6x_1+0.4x_2-x_5+x_6=6$

Now for the tableau (my apologies for not having it formatted just like it is in the examples I use in class):

		Coefficient of:								
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	$-1.1M+0.4$	$-0.9M+0.5$	0	0	M	0	$-12M$	
x_3	1	0	0.3	0.1	1	0	0	0	2.7	
x_4	2	0	0.5	0.5	0	1	0	0	6	
x_6	3	0	0.6	0.4	0	0	-1	1	6	

Iteration 1: find the pivot column - we're looking at our Eq. 0 here and the coefficients across the various x values. Since M is such a large factor, we really only consider the M coefficients. Remember, we're looking for the negative coefficient with the largest absolute value. The values are:

x_1 : $-1.1M-2$
x_2 : $-0.5M-5$

This means that x_1 has the negative coefficient with the largest absolute value. x_1 is our entering variable (our pivot column).

Next we find the pivot row by comparing the right side entry divided by the strictly positive coefficients in the pivot column. The smallest of these ratios is our leaving variable (pivot row). Those ratios are noted in the ratio column.

		Coefficient of:								
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	$-1.1M+0.4$	$-0.9M+0.5$	0	0	M	0	$-12M$	
x_3	1	0	0.3	0.1	1	0	0	0	2.7	9
x_4	2	0	0.5	0.5	0	1	0	0	6	12
x_6	3	0	0.6	0.4	0	0	-1	1	6	10

Thus, our pivot row is x_3 and that's our leaving variable. Next step: get our pivot number to 1 by dividing by 0.3 (entries changed noted in red).

		Coefficient of:								
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	$-1.1M+0.4$	$-0.9M+0.5$	0	0	M	0	$-12M$	
x_3	1	0	**1**	**1/3**	**10/3**	0	0	0	**9**	
x_4	2	0	0.5	0.5	0	1	0	0	6	
x_6	3	0	0.6	0.4	0	0	-1	1	6	

Now we eliminate x_1 from Eq 0, Eq 2, and Eq 3 by performing row operations. Since the coefficient of x_1 in Eq 0 is $-1.1M+0.4$ we multiply the entirety of Eq 1 by $1.1M-0.4$ and add it to Eq 0. Basically:

Eq 0 : $(-1.1M+0.4)x_1+(-0.9M+0.5)x_2+Mx_5=-12M$
PLUS: $(1.1M-0.4)x_1+(11/30M-2/15)x_2+(11/3M-4/3)x_3=9.9M-3.6$

Which leads to:

		Coefficient of:								
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	**0**	$\mathbf{-16/30M+11/30}$	$\mathbf{11/3M-4/3}$	0	M	0	$\mathbf{-2.1M-3.6}$	
x_3	1	0	1	1/3	10/3	0	0	0	9	
x_4	2	0	0.5	0.5	0	1	0	0	6	
x_6	3	0	0.6	0.4	0	0	-1	1	6	

And, we have to do the same thing to Eq2 and Eq3, multiplying the entirety of Eq1 by 1/2 and subtracting from Eq 2 and multiplying Eq1 by 3/5 and subtracting from Eq 3. Basically:

Eq 2: $1/2x_1+1/2x_2+x_4=6$
MINUS: $1/2x_1+1/6x_2+5/3x_3=4.5$

Eq 3: $3/5x_1 + 2/5x_2 - x_5 + x_6 = 6$
MINUS: $3/5x_1 + 1/5x_2 + 2x_3 = 5.4$

Which leads to:

		Coefficient of:								
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	0	$-16/30M + 11/30$	$11/3M - 4/3$	0	M	0	$-2.1M - 3.6$	
x_1	1	0	1	1/3	10/3	0	0	0	9	
x_4	2	0	**0**	**1/3**	**-5/3**	1	0	0	**3/2**	
x_6	3	0	**0**	**1/5**	**-2**	0	-1	1	**3/5**	

Since we still have a negative coefficient in Eq 0, we continue onward with iteration 2. First, since we have only one negative coefficient in Eq 0, we know that x_2 will be our entering variable. We have three strictly positive coefficients in the x_2 column, so we have to look at the ratios:

		Coefficient of:								
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	0	$-16/30M + 11/30$	$11/3M - 4/3$	0	M	0	$-2.1M - 3.6$	
x_1	1	0	1	1/3	10/3	0	0	0	9	27
x_4	2	0	0	1/3	-5/3	1	0	0	3/2	4.5
x_6	3	0	0	1/5	-2	0	-1	1	3/5	3

Eq 3 is our smallest ratio, so we have that as our leaving variable. Our pivot number is 1/5, so we need to multiply the entire row by 5 to get the coefficient to 1. The tableau looks like (values changed in red):

		Coefficient of:								
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	0	$-16/30M + 11/30$	$11/3M - 4/3$	0	M	0	$-2.1M - 3.6$	
x_1	1	0	1	1/3	10/3	0	0	0	9	
x_4	2	0	0	1/3	-5/3	1	0	0	3/2	
x_6	3	0	0	**1**	**-10**	0	**-5**	**5**	**3**	

Then we perform row operations as above, adding to Eq 0, and subtracting from Eq 1 and 2:

Eq 0: $(-16/30M + 11/30)x_2 + (11/3M - 4/3)x_3 + Mx_5 = -2.1M - 3.6$
PLUS: $(16/30M - 11/30)x_2 - (16/3M - 11/3)x_3 - (8/3M - 11/6)x_5 + (8/3M - 11/6)x_6 = 8/5M - 11/10$

and

Eq 1: $x_1 + 1/3x_2 + 10/3x_3 = 9$
MINUS: $1/3x_2 - 10/3x_3 - 5/3x_5 + 5/3x_6 = 1$

and

Eq 2: $1/3x_2 - 5/3x_3 + x_4 = 3/2$
MINUS: $1/3x_2 - 10/3x_3 - 5/3x_5 + 5/3x_6 = 1$

Which leaves our tableau (values changed in red):

		Coefficient of:								
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	0	**0**	**-5/3M+7/3**	0	**-5/3M+11/6**	**8/3M -11/6**	**-1/2M-47/10**	
x_1	1	0	1	**0**	**20/3**	0	**5/3**	**-5/3**	**8**	
x_4	2	0	0	**0**	**5/3**	1	**5/3**	**-5/3**	**1/2**	
x_2	3	0	0	1	-10	0	-5	5	3	

We still have negative coefficients in Eq 0, so we continue with iteration 3. We have two coefficients with the same M coefficient. It doesn't really matter which one of these we'll choose (the constant is discarded as above) so we'll choose x_5 to be our pivot column. Note, we could choose x_3 and we would still get to a correct solution. As before, we take a look at the positive coefficients in the x_5 column and get the ratios:

		Coefficient of:								Right Side	Ratio
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6		Right Side	Ratio
Z	0	-1	0	0	-5/3M+7/3	0	-5/3M+11/6	8/3M -11/6		-1/2M-47/10	
x_1	1	0	1	0	20/3	0	5/3	-5/3		8	24/5
x_4	2	0	0	0	5/3	1	5/3	-5/3		1/2	3/10
x_2	3	0	0	1	-10	0	-5	5		3	

The smallest ratio here belongs to the x_4 row and as such, we have 5/3 as our pivot value. Multiplying Eq 2 by 3/5 gets our pivot value to 1 and a new table:

		Coefficient of:							Right Side	Ratio
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	0	0	-5/3M+7/3	0	-5/3M+11/6	8/3M -11/6	-1/2M-47/10	
x_1	1	0	1	0	20/3	0	5/3	-5/3	8	
x_4	2	0	0	0	**1**	**3/5**	**1**	**-1**	**3/10**	
x_2	3	0	0	1	-10	0	-5	5	3	

Again with the row operations, adding to Eq 0 and Eq 3, subtracting from Eq 1.

Eq 0: $(-5/3M + 7/3)x_3 - (5/3M - 11/6)x_5 + (8/3M - 11/6)x_6 = -1/2M - 47/10$
PLUS: $(5/3M - 11/6)x_3 + (M - 11/10)x_4 + (5/3M - 11/6)x_5 - (5/3M - 11/6)x_6 = 1/2M - 11/20$

and

Eq 1: $x_1 + 20/3x_3 + 5/3x_5 - 5/3x_6 = 8$
MINUS: $5/3x_3 + x_4 + 5/3x_5 - 5/3x_6 = 1/2$

and

Eq 3: $x_2 - 10x_3 - 5x_5 + 5x_6 = 3$
PLUS: $5x_3 + 3x_4 + 5x_5 - 5x_6 = 3/2$

Leaving us our new table:

		Coefficient of:							Right Side	Ratio
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side	Ratio
Z	0	-1	0	0	**0.5**	**M - 11/10**	**0**	**M**	**-21/4**	
x_1	1	0	1	0	**5**	**-1**	**0**	**0**	**15/2**	
x_5	2	0	0	0	1	3/5	1	-1	3/10	
x_2	3	0	0	1	**-5**	**3**	**0**	**0**	**9/2**	

Since we have positive coefficients in our Eq 0, we know we're at an optimal solution. Note, that since Z has a -1 coefficient, we need to negate the right side value. Thus, we get $Z = 5.25$, $x_1 = 7.5$, $x_2 = 4.5$.